Exercise 2: Control Issues

Task 1: Controlling the Global Skew

We left open how a small global skew is achieved by the GCS algorithm from the lecture. In this exercise, you modify the algorithm such that this is the case, without changing its properties regarding the local skew.

a) Add the condition that any node $v \in V$ satisfying $L_v(t) = \max_{w \in V}\{L_w(t)\}$ is in slow mode at time t and determine a suitable trigger condition. Show that your trigger condition is not in conflict with **FT**.

b) Apply the techniques used in the (refined) Max Algorithm to maintain an estimate $M_v(t)$ of the largest clock value throughout the system at each $v \in V$. Show that $\max_{v \in V}\{L_v(t)\} \geq M_v(t) \geq \max_{v \in V}\{L_v(t)\} - \mathcal{G}_{\max}$ for some $\mathcal{G}_{\max}$. (Hints: Make minimal modifications to the Max Algorithm, so that the reasoning does change very little. This way, you can argue that the proof of the bound is analogous. Note that you need to be slightly more careful regarding the rate at which nodes increase the estimates when $L_v(t) < M_v(t)$: use rate $h_v/\vartheta \leq 1$. You should obtain $\mathcal{G}_{\max} = ((\vartheta - 1/\vartheta)T + (\vartheta - 1)d + u)D$.)

c) Show that $L_v(t) = \min_{w \in V}\{L_w(t)\}$ implies that v does not satisfy **ST** at time t.

d) Assume that $\sigma = \mu/(\vartheta - 1) > 1$ and that $\max_{v \in V}\{H_v(0)\} \leq \mathcal{G}_{\max}$. Add the condition that any node $v \in V$ satisfying that $L_v(t) < M_v(t)$ and **ST** does not hold at time t is in fast mode. Conclude that the modified algorithm has global skew $\mathcal{G} \leq \mathcal{G}_{\max}$ and still obeys **FC** and **SC**. What is the resulting local skew, provided that $\max_{\{v,w\} \in E}\{H_v(0) - H_w(0)\} \leq \delta$?

Task 2: Controlling Uncertainty

In the lecture, we assumed that $v \in V$ has an estimate $\tilde{L}_w$ of the logical clock L_w of each of its neighbors $w \in N_v$, satisfying that $L_w(t) - \delta < \tilde{L}_w(t) \leq L_w(t)$ at all times t. In this exercise, you determine δ for a straightforward way of deriving such an estimate. You may assume that $\max_{\{v,w\} \in E}\{H_v(0) - H_w(0)\} \leq \delta - (\vartheta(1+\mu) - 1/\vartheta)d$ throughout this exercise and that $\vartheta \in \mathcal{O}(1)$.

a) Suppose $w \in V$ sends a message with its current logical clock value whenever $H_v(t) = kT$ for some $k \in \mathbb{N}$, and also at time 0. Determine a (good) estimate $\tilde{L}_w(t)$ that $v \in V$ can compute based on this information. Bound the resulting δ. (Hint: It's ok to be a bit sloppy with lower order terms or constant factors, as long as you get the asymptotics right.)

b) For fixed values of all other parameters, determine a choice of μ asymptotically minimizing our upper bound on the local skew (i.e., up to constant factors). (Hint: Argue that $\delta \in \Omega(\mathcal{G})$ implies that the upper bound is trivial and that it doesn't matter (asymptotically) to choose μ to be at least $\max\{u/(T+d), 8(\vartheta - 1)\}$. Having ruled out these corner cases, check how the bound changes if a value of μ satisfying these constraints is doubled.)

c) For this method of determining estimates, the asymptotically optimal choice of μ you computed, and the global skew bound you obtained in the first exercise, determine the bound on the local skew as function of T (use the same value of T for global and local estimates).

Task 3*: Control Right from the Beginning

So far, we've been sweeping the initialization process under the rug. Clearly, it's unrealistic to assume that all nodes start executing the algorithm precisely at time 0. This would require perfect synchronization! Instead, we now assume that nodes can spontanteously wake up and execute the algorithm at any time. However, receiving a message wakes them up, too. The hardware clock of a node is 0 at the time when it wakes up. W.l.o.g., assume that at least one node wakes up at time 0.

a) Initialize the network by flooding, i.e., on wake-up, a node broadcasts a message to all its neighbors. Adapt the clock estimation technique from Task 2 to account for the modified initialization.

b) Extend the hardware clock functions, logical clock functions, and clock estimates to be defined from time 0 on such that (i) $1 \leq h_v(t) \leq \vartheta$ for all t, (ii) $L_v(t) = H_v(t)$ at times t when v has not yet woken up, (iii) $L_w(t) - \delta < \tilde{L}_w(t) \leq L_w(t)$ at all times t, and (iv) node v is in slow mode at times t when it has not woken up yet according to the (modified) GCS algorithm.

c) Convince Saeed and your fellow students that this approach yields the same skew bounds you computed in Tasks 1 and 2!