Exercise 3: Gradients are Hard

Task 1: No Uncertainty, still no Knowing the Time

In some networks, frequent communication is too expensive, for instance in terms of energy. This is especially true in wireless sensor networks, where successfully sending a message is a challenge in itself. In such networks, a fairly complicated scheduler may be in place. We model this by taking away the control over the message schedule from our algorithm designer: An adversary decides when messages are sent, with the only condition that a node broadcasts its current logical clock value at least every T time. The uncertainty and even message transmission times may be small in comparison to the influence of this issue; for simplicity, assume that messages arrive instantaneously, i.e., $d = u = 0$.

a) Show that you can choose message sending times and hardware clock rates such that you can introduce a skew of $\Omega((\vartheta - 1)Td(v, w))$ between the hardware clocks of nodes v and w before they notice (assuming that $\vartheta \in \mathcal{O}(1)$). Aim for a statement that can replace Lemma 3.2! You are free to show the claim for bipartite graphs only.

b) Use this to prove a theorem analogous to Theorem 3.1 in this setting! (Discussing the changes that have to be made to the proof of Theorem 3.1 suffices.)

c) How well does the GCS algorithm do in this scenario?

d) Show that one can do *much* better if the message schedule is under one's control!

Task 2: No Fast Edge Insertion

Suppose that new edges may be added to the graph at arbitary times. The endpoints of the edges will know that the edge is new, and seek to ensure a small local skew also on such edges.

a) Show a worst-case lower bound of $(\mathcal{G} - \mathcal{L})/\mu$ on the time required to achieve a skew of at most $\mathcal{L}$ on a new edge. (Assume that there was no prior edge insertion.)

b) Show that, in the worst case and even with unbouded logical clock rates, $\Omega((d - u)uD/\mathcal{L})$ time is required to achieve a skew of at most $\mathcal{L}$ on a new edge, if the skew on the edges that were fully integrated must not exceed $\mathcal{L} \ll uD$. Here, D denotes the diameter of the graph before the (single) additional edge is inserted. (Hint: Consider an execution with skew $\Omega(uD)$ between $v, w \in V$ at some time $t \geq (d - u)uD/\mathcal{L}$ and insert $\{v, w\}$ at time $t - \Theta((d - u)uD/\mathcal{L})$.)