Exercise 9: Stabilization

Task 1: Recovering Reasonable Constraints

In this exercise, we want to get a better handle on the set of constraints from the lecture:

$$\mathcal{S}(1) \geq 2\left(\delta + \left(1 - \frac{1}{\vartheta}\right)T\right) \tag{1}$$

$$\frac{R^-}{\vartheta} \geq \sigma_h + \vartheta\mathcal{S}(1) + d \tag{2}$$

$$\frac{B_2}{\vartheta} > \sigma_h + R^+ + T + 2\mathcal{S}(1) \tag{3}$$

$$B_1 > \sigma_h + R^+ \tag{4}$$

$$B_3 > R^+ + (M-1)(T + \mathcal{S}(1)) + (\vartheta + 1)\mathcal{S}(M) + \sigma_h \tag{5}$$

$$B_2 \leq \frac{R^-}{\vartheta} + (M-1)\left(\frac{T}{\vartheta} - \mathcal{S}(1)\right) + \mathcal{S}(M) \tag{6}$$

$$\frac{R^+}{\vartheta} \geq (\vartheta + 1)\mathcal{S}(M) + \sigma_h \tag{7}$$

$$2(\mathcal{S}(1) - \mathcal{S}(M)) \geq \sigma_h \tag{8}$$

a) Choose R^- tight according to (2) and R^+/ϑ equal to the r.h.s. of (7) plus d, respectively. Show that for these choices (1) and (8) hold. (Hint: Use these equalities to express $\mathcal{S}(1)$ in terms of $\mathcal{S}(M)$ and other terms not involving R^+ or R^-.)

b) Fix these choices and consider the remaining inequalities. Which of the terms on the r.h.s. of the inequalities are in $\mathcal{O}(\sigma_h + d)$? (Hint: Recall that $\vartheta \in \mathcal{O}(1)$.)

c) In addition, suppose now that for $\alpha > 1$, we have that $B_3 = \alpha B_2 = \alpha^2 B_1$ and can choose $B_1 > 0$ arbitrarily. For which values of α can you choose M so that the system of inequalities is satisfied?

Task 2: If it Were so Simple...

a) Modify the Srikanth-Toueg algorithm so that its pulses can be triggered by an external NEXT signal in the vein of Definition 9.7. (Hint: Add a second, smaller timeout to the transition from READYto PROPOSE.)

b) What choices of B_1, B_2, and B_3 can this solution support?