Karl Bringmann and Marvin Künnemann **Summer 2019**

Fine-Grained Complexity Theory, Exercise Sheet ◯◯

www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/summer19/fine-complexity/

Total Points: **0** Due: The Day before Yesterday

You are allowed to collaborate on the exercise sheets, but you have to write down a solution on your own, ***using your own words****. Please indicate the names of your collaborators for each exercise you solve. Further, cite all external sources that you use (books, websites, research papers, etc.).*

You need to collect at least 50% of all points on exercise sheets to be admitted for the exam.

Exercise 1

For each of the following problems, determine whether it can be solved in strongly subquadratic time (that is in time $O(n^{2-\varepsilon})$ for some $\varepsilon > 0$).
Prove your claims by giving either an algorithm running in strongly subquadratic time or a hardness proof that rules out such an algorithm under some conjecture discussed in the course.

a) **Longest Palindromic Subsequence**: Given a string S of length n, find the longest subsequence that is a palindrome (that is, a sequence of characters which reads the same backwards and forwards).

b) **Non-Dominating Vectors (Constant Dimension)**: Given a set $A \subseteq \mathbb{Z}^d$ of n integer vectors, $d = O(1)$, compute the set $A' \subseteq A$ of non-dominated vectors.
(A vector $a \in A$ dominates another vector $a' \in A$ if $a_i \geq a'_i$ for all $1 \leq i \leq d$ and $a \neq a'$.)

c) **Non-Dominating Vectors (Low Dimension)**: Given a set $A \subseteq \mathbb{Z}^d$ of n integer vectors, $d = \log^3 n$, compute the set $A' \subseteq A$ of non-dominated vectors.

Exercise 2

The **Minimum Consecutive Sums Problem** is defined as follows:

MCSP: Given n integers $x_1, x_2, \ldots, x_n$, determine for any $1 \leq k \leq n$ the minimal sum of any k consecutive of these integers, that is, compute for any $1 \leq k \leq n$ the number

$$\min\{x_i + \ldots + x_{i+k-1} \mid 1 \leq i \leq n-k+1\}.$$

Prove that **(min,+)–Convolution** and **MCSP** are equivalent in the following sense:

$$(\textbf{MCSP}, n^2) \leq_{fgr} ((\textbf{min,+})\textbf{–Convolution}, n^2) \leq_{fgr} (\textbf{MCSP}, n^2).$$

Exercise 3

Recall the **Longest Common Subsequence** problem from the lecture:

Longest Common Subsequence: Given string S and T of length n each, compute the length $L = L(S,T)$ of the longest string C that is a subsequence of both S and T.

Recall that we proved an $n^{2-o(1)}$ lower bound for this problem under **SETH** during the lecture.

a) Given t instances $(S_1, T_1), \ldots, (S_t, T_t)$ of the **LCS** problem, show that we cannot compute the maximum LCS of the instances, that is $\max_i L(S_i, T_i)$, in time $O((tk^2)^{1-\varepsilon})$, for any $\varepsilon > 0$ (conditioned on **SETH**), where each string has length at most k.

b) Prove an $(m \cdot L)^{1-o(1)}$ lower bound for the **LCS** problem (conditioned on **SETH**), where $L = L(S,T)$ is the length of the longest common subsequence of S and T and $m = \min\{|S|, |T|\}$ is the length of the shorter string.