Dániel Marx and Pranabendu Misra

Summer 2020

Parameterized Algorithms, Exercise Sheet 2

www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/summer20/parameterized-algorithms/

Total Points: 50

Due: Friday, **May 29**, 2020

You are allowed to collaborate on the exercise sheets, but you have to write down a solution on your own, ***using your own words****. Please indicate the names of your collaborators for each exercise you solve. Further, cite all external sources that you use (books, websites, research papers, etc.). You need to collect at least 50% of all points on exercise sheets to be admitted to the exam.*
Please send your solutions directly to Philip (wellnitz@mpi-inf.mpg.de).

Exercise 1 — 5 + 5 points

The Feedback Vertex Set on Tournaments (FVST) problem is defined as follows: The input are a tournament D and an integer k, and the task is to decide if there is a subset of vertices $X \subseteq V(D)$ such that $D \setminus X$ is an acyclic digraph.

(a) Show that D contains a cycle if and only if D contains a cycle of length 3.
(b) Design a $c^k \cdot \mathsf{poly}(n)$ time algorithm for FVST using *Iterative Compression*.

Exercise 2 — 5 + 5 points

(a) Recall the Set Cover problem, and its dynamic programming algorithm (parameterized by size of the universe) from the lecture. Prove the correctness of the *Recursive Step* of the algorithm.
(b) Recall the Steiner Tree problem, and the dynamic programming algorithm (parameterized by the number of terminal vertices) from the lecture. Consider the *weighted* version of the problem, where there is a weight function $w : V(D) \to \mathbb{R}^+$, and the objective if the find the minimum weight Steiner tree. Obtain an algorithm for Weighted Steiner Tree parameterized by the number of terminals.

Exercise 3 — 5 + 5 points

Suppose that we know that parameterized problem B can be solved in time $3^k \cdot n^{O(1)}$. Furthermore, there is a parameterized reduction from problem A to problem B that runs in polynomial time. What running time bound we can give for problem A if we know that the reduction creates new instances with parameter

(a) at most $4k$,
(b) at most $2k^2$?

Exercise 4 — 10 points

Either show that the following problem is FPT or show that it is W[1]-complete.

Distance-4 Independent Set	
Input:	A graph G, an integer k
Parameter:	k
Question:	Is there a set S of k vertices in G such such that the distance of any two vertices in S is at least 4?

Exercise 5 — 10 points

In the Set Packing problem the input consists of a family $\mathcal{F}$ of subsets of a finite universe U and an integer k, and the question is whether one can find k pairwise disjoint sets in F. Prove that Set Packing is W[1]-hard when parameterized by k.