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Summer 2020

Parameterized Algorithms, Exercise Sheet 3

www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/summer20/parameterized-algorithms/

Total Points: 50

Due: Friday, **June 12**, 2020

You are allowed to collaborate on the exercise sheets, but you have to write down a solution on your own, ***using your own words****. Please indicate the names of your collaborators for each exercise you solve. Further, cite all external sources that you use (books, websites, research papers, etc.). You need to collect at least 50% of all points on exercise sheets to be admitted to the exam.*
Please send your solutions directly to Philip (wellnitz@mpi-inf.mpg.de).

Exercise 1 — 10 points

In the Point Line Cover problem, we have n points on the plane, and the goal is to cover these points with at most k lines; the parameter is k. Design a polynomial kernel for Point Line Cover.

Exercise 2 — 10 points

A graph is a *cluster graph* if it is the disjoint union of cliques. Given a graph G and integer k, consider the problem of deciding if there is a subset S of k vertices such that $G \setminus S$ is a cluster graph. Obtain a polynomial kernel for this problem.

Exercise 3 — 5 + 10 points

In the Connected Vertex Cover (CVC) problem, given a graph G and integer k the task is to find a vertex cover S such that $G[S]$ is connected and $|S| \leq k$. Consider CVC parameterized by k.

(a) Show that CVC admits an *exponential* kernel with size $O(k^2) + 2^k$.

(b) Show that CVC admits a polynomial kernel when the input graph has no cycles of length 4 or smaller.

Exercise 4 — 5 points

In the Max Leaf Subtree problem, given a graph G and integer k the goal is to find a sub-tree with at least k leaves. Show that this problem does not admit a polynomial kernel.

Exercise 5 — 10 points

Show that the Connected Vertex Cover problem parameterized by the solution size k, does not admit a polynomial kernel. You may assume that Set Cover parameterized by the size of the universe doesn't admit a polynomial kernel.