Dániel Marx and Pranabendu Misra **Summer 2020**

Parameterized Algorithms, Exercise Sheet 5

www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/summer20/parameterized-algorithms/

Total Points: 50 Due: Friday, **July 10**, 2020

You are allowed to collaborate on the exercise sheets, but you have to write down a solution on your own, ***using your own words****. Please indicate the names of your collaborators for each exercise you solve. Further, cite all external sources that you use (books, websites, research papers, etc.). You need to collect at least 50% of all points on exercise sheets to be admitted to the exam.*
Please send your solutions directly to Philip (wellnitz@mpi-inf.mpg.de).

Exercise 1 — 4 + 4 + 2 points

Let G be an undirected graph, and let s and t be two vertices of G.

(a) What is the maximum number of important (s,t)-cuts of size *exactly 1* in G?

(b) What is the maximum number of important (s,t)-cuts of size *exactly 2* in G?

(c) What is the maximum number of important (s,t)-cuts of size *at most 2* in G?

Justify your answers.

Exercise 2 — 10 points

Let G be an undirected graph, and let s and t be two vertices of G. Is it true that the number of important (s,t)-cuts of size at most k is the same as the number of important (t,s)-cuts of size at most k? Justify your answer.

Exercise 3 — 10 points

Give a parameterized reduction from Undirected Multiway Cut to Directed Multiway Cut that does not change the parameter. That is, given an instance (G,T,k) of Undirected Multiway Cut, construct an equivalent instance (G',T',k) of Directed Multiway Cut.

Exercise 4 — 10 points

Given an undirected graph G, a subset T of vertices, and two integers k and ℓ, the Short Multiway Cut problem asks for a set S of at most k edges such that the graph $G-S$ contains no path of length at most ℓ between two distinct vertices of T. Show that the problem is FPT with combined parameters k and ℓ.

Exercise 5 — 10 points

Given a directed graph G and an integer k, the Reversal to DAG problem asks for a set S of at most k edges such that if we reverse the orientation of the edges in S, then we get a directed acyclic graph. Show that Reversal to DAG is FPT parameterized by k.