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Geometric algorithms with limited resources, Exercise Sheet 1

https://www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/summer21/geometric-algorithms-with-limited-resources

Total Points: 40 Due: 8am, Thursday, **April 29th**, 2020

You are allowed to collaborate on the exercise sheets, but you have to write down a solution on your own, ***using your own words****. Please indicate the names of your collaborators for each exercise you solve. Further, cite all external sources that you use (books, websites, research papers, etc.). You need to collect at least 50% of all points on exercise sheets to be admitted to the exam.*

Exercise 1 — **4** points

Give a 2-pass algorithm that reports 3 extreme points of the convex hull using $O(1)$ space. (Note that the rightmost and bottommost points of a point set might coincide.)

Exercise 2 — **5** points

Given $s > 0$ and a simple non-convex polygon (non self-intersecting, hole-free) as the cyclic sequence of its vertices, compute the vertices of its convex hull in $O(s)$ available space, $O(n^2/s)$ time, and $O(\lceil n/s \rceil)$ passes. (You may assume that the polygon's vertices are in general position.)

Exercise 3 — **4** + **7** points

a) Suppose that we have a stream of $n-2$ distinct integers among $\{1,\dots,n\}$. Give a streaming algorithm on a Word RAM that reports the two missing integers using $O(1)$ space. (Recall that Word RAM machines can store integers of up to $O(\log n)$ bits).

b) Suppose that we have a stream of n distinct integers among $\{1,\dots,2n\}$. Give a streaming algorithm on a Real RAM that reports the n missing integers using $O(1)$ space and $\widetilde{O}(n)$ time. (Recall that Real RAM can store and do basic arithmetic operations $(+,-,\cdot,/)$ on arbitrary real numbers. However, it does not have floor, rounding and modulo operations.)

Exercise 4 — **5** points

Prove the following lemma regarding the algorithm that estimates the number of connected components in a graph (presented in lecture 2): It holds that:

$$\Pr[|\hat{c}-\tilde{c}| > \epsilon n/2] \leq 1/4$$

Exercise 5 — **4+4+7** points

The goal of this problem is to find a sub-linear time algorithm for approximating the weight of the minimum spanning tree (MST) in a graph. We are given a connected graph $G=(V,E)$. This graph is represented as an adjacency list and there are $n=|V|$ nodes. Each edge (i,j) has integer weight $w_{ij} \in \{1,\dots,w\} \cup \infty$. We will need the following definitions:

- $G^{(i)} = (V, E(i))$ where $E(i) = \{(u,v) | w_{uv} \in \{1,...,i\}\}$.
- $C^{(i)}$ is the number of connected components in $G^{(i)}$.

a) You are given an algorithm A for a decision problem (i.e., answer for each input is either 0 or 1), that runs in time $T(n)$ on inputs of size n, with probability of error $1/4$. Show how to convert it into a new algorithm B that runs in time $O(T(n)\log 1/\beta)$ with probability of error at most β. (Hint: run A $O(\log 1/\beta)$ times and take the "majority", i.e., the most common, answer. Use Chernoff bounds to show that the correct answer is highly likely to be the output.)

b) Let M denote the minimum spanning tree of G. Show the following:

$$M = n - w + \sum_{i=1}^{w-1} C^{(i)}$$

c) Show that by estimating each $C^{(i)}$ using the algorithm from lecture 2 with parameter $\epsilon' = \frac{\epsilon}{2w}$ and amplified according to (a) to achieve an error probability $\beta = \frac{1}{10w}$, we can compute an $(1+\epsilon)$- approximation of the MST of G with probability at least $9/10$. What is the running time of your algorithm?