Sándor Kisfaludi-Bak and Themis Gouleakis **Summer 2021**

Geometric algorithms with limited resources, Exercise Sheet 3

https://www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/summer21/geometric-algorithms-with-limited-resources

Total Points: 40 Due: 8am, Thursday, **May 27th**, 2021

*You are allowed to collaborate on the exercise sheets, but you have to write down a solution on your own, **using your own words**. Please indicate the names of your collaborators for each exercise you solve. Further, cite all external sources that you use (books, websites, research papers, etc.). You need to collect at least 50% of all points on exercise sheets to be admitted to the exam.*

Exercise 1 — **7** points

Show that the expected running time of any comparison-based Las Vegas sorting algorithm is $\Omega(n \log n)$.

Exercise 2 — **4** points

Let P and Q be disjoint convex polytopes in $\mathbb{R}^3$, and suppose that $p \in P$ and $q \in Q$ satisfies $\operatorname{dist}(p, q) = \min_{x\in P, y\in Q} \operatorname{dist}(x, y)$. Let H_p be a plane perpendicular to pq through p. Show that $H_p \cap P$ is either a vertex, an edge or a face of P.

Exercise 3 — **9** points

Let P and Q be two disjoint convex polygons given by their cyclic list of vertices, stored in an array arbitrarily. Find their shared outer tangents in $O(\sqrt{n})$ expected time.

Exercise 4 — **7+2** points

(a) Show that for any integer d there exists a convex polygon C in the plane such that the family of all rotations of C around the origin has VC dimension at least d.
Hint: Consider constructing a convex polygon, which is inscribed in the unit circle.

(b) How many vertices does the polygon have (as a function of d)?

Exercise 5 — **3+8** points

As a reminder from the lecture, a *sleeve* between 2 pixels p_1, p_2 of an image is the union of all the imprints of line segments starting in p_1 and ending in p_2.

(a) Show that every sleeve between 2 pixels contains at most 3 pixels in each row or at most 3 pixels in each column.

(b) Show that the set of all line imprints containing p_1 and p_2 has constant VC dimension.
Hint: Partition the sleeve into 3 sets of pixels and argue that only a constant number of pixels from each set can be contained in a shattered set.