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Exercises for Approximation Algorithms

www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/winter15/approx

Tutorials: Andreas Schmid

Exercise Sheet 5 Due: **22.1.2016**

Your homework must be handed in on ***Friday*** *at the beginning of the tutorial.*

You need to collect at least 50% of all points over all exercise sheets. You are allowed to work on these exercises in groups but every student has to hand in his/her own write-up.

Exercise 1 (*10 points*)

Consider the following modification to the facility location problem. Define the cost of connecting client j to facility i to be c_{ij}^2. The c_{ij}'s satisfy the triangle inequality but the new connection costs of c_{ij}^2 do not. Show that the approximation algorithm shown in the lecture can perform arbitrary bad in this setting.

Exercise 2 (*10 points*) [De Berg et.al. Exercise 2.14]

Let S be a set of n disjoint line segments in the plane, and let p be a point not on any of the line segments of S. We wish to determine all line segments of S that p can see, that is, all line segments of S that contain some point q so that the open segment $\overline{pq}$ does not intersect any line segment of S. Give an $O(n \log n)$ time algorithm for this problem that uses a rotating half-line with its endpoint at p.

Exercise 3 (*8 points*) [De Berg et.al. Exercise 4.16]

On n parallel railway tracks n trains are going with constant speeds $v_1, v_2, \ldots, v_n$. At time $t = 0$ the trains are at positions $k_1, k_2, \ldots, k_n$. Give an $O(n \log n)$ algorithm that detects all trains that at some moment in time are leading.

Exercise 4 (*12 points*) [De Berg et.al.]

a) In the lecture it was shown that for $n \geq 3$, the number of edges in the Voronoi diagram is at most $3n - 6$. In addition it is known that the number of vertices is at most $2n - 5$. Show that this implies that the average number of vertices of a Voronoi cell is less than six.

The following parts of the exercise are about Fortunes algorithm.

b) Give an example where the parabola defined by some site p_i contributes more than one arc to the beach line. Can you give an example where it contributes a linear number of arcs?

c) Give an example of six sites such that the plane sweep algorithm encounters the six site events before any of the circle events. The sites should lie in general position: no three sites on a line and no four sites on a circle.

d) Do the breakpoints of the beach line always move downwards when the sweep line moves downwards? Prove this or give a counterexample.

Exercise 5 (***BONUS*** *10 points*) [Shmoys-Williamson Exercise 9.2]

Given a minimization problem with instances I and a local search algorithm A, let S_i be the set of local optimal solutions for instance $i \in I$. For a solution $s \in S_i$ we denote by $c(s)$ its cost. We define the locality gap α_A of A as the largest possible ratio between a local optimal solution $s \in S_i$ and a global optimum solution s_i^* for all $i \in I$.

$$\alpha_A = \max_{i \in I}\{\max_{s \in S_i}\{\frac{c(s)}{c(s_i^*)}, \frac{c(s_i^*)}{c(s)}\}\}$$

In this exercise we want to find the locality gap of the local search algorithm for the facility location problem shown in the lecture.

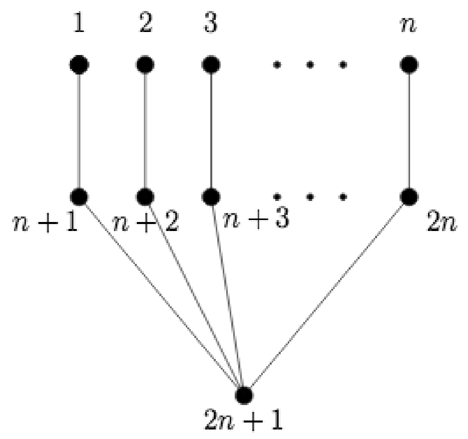

Figure 1: Instance for Exercise 5 showing a bad locality gap for the local search algorithm shown in class

Consider the instance shown in Figure 1, where the facilities $F = \{1, \ldots, n, 2n+1\}$, and the clients $D = \{n+1, \ldots, 2n\}$. The cost of each facility $1, \ldots, n$ is 1, while the cost of facility $2n+1$ is $n-1$. The cost of each edge in the figure is 1, and the assignment cost c_{ij} is the shortest path distance in the graph between $i \in F$ and $j \in D$. Use the instance to show that the locality gap is at least $3 - \epsilon$ for any $\epsilon > 0$.