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Exercises for Algorithms and Data Structures

http://www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/winter16/algorithms-and-data-structures/

Exercise Sheet 9 Due: **16.1.2017**

The homework must be handed in on Monday before the lecture. You may collaborate with other students on finding the solutions for this problem set, but every student must hand in a writeup in their own words. We also expect you to state your collaborators and sources (books, papers, course notes, web pages, etc.) that you used to arrive at your solutions.

You need to collect at least 50% of all points on all exercise sheets to be admitted to the final exam.

Whenever you are asked to design an algorithm in this exercise sheet you have to give a proof of its correctness as well as an asymptotic upper bound on its worst case running time.

Exercise 1 (*10 points*)

The teams *Bedigliora*, *Caslano*, *Novaggio*, *Pura* and *Sessa* form a league. In every season, every team plays six matches against every other team. In each match, one point will be given to the winner (and there is always a unique winner). The current table of points is given to the right. Bedigliora has 5 matches remaining against Caslano, 3 matches remaining against Sessa, and 6 matches remaining against Novaggio. Caslano has 2 matches remaining against Sessa and 5 matches remaining against Novaggio. Sessa has 3 matches remaining against Novaggio. (The remaining games of Pura can be infered indirectly.)

Team	Points
Caslano	8
Novaggio	7
Bedigliora	6
Sessa	3
Pura	0

If Pura hires a superstar right now to be better than all other teams – is it possible for Pura to win the cup? *Is there a possible outcome of all remaining games such that Pura has most points at the end of the season (possibly sharing with another team)?*

Hint: Model (parts of) the question by a flow problem.

Exercise 2 (*10 points*)

To represent a positive integer n in base B, with $B \geq 2$, we write $n = n_0 + n_1 B + \ldots + n_k B^k$

for some $k \geq 1$ and $n_i \in \{0, \ldots, B-1\}$ for all i.

You are given numbers $\tilde{n}_0, \ldots, \tilde{n}_k \in \{0, \ldots, B^2 - 1\}$ such that $n = \tilde{n}_0 + \tilde{n}_1 B + \ldots \tilde{n}_k B^k$. Note that this is not yet a valid base B representation, since the coefficients can be larger than $B - 1$. Design an algorithm converting $\tilde{n}_0, \ldots, \tilde{n}_k$ into a valid base B representation of n. Aim at time complexity $O(k)$ (assuming that integers up to $O(B^2)$ fit into one memory cell).

Exercise 3 (*10 points*)

A polynomial in x and y of degree d can be written as $\sum_{i+j \leq d} \alpha_{i,j} \cdot x^i y^j$, where the sum is over all $i, j \in \{0, \ldots, d\}$ with $i + j \leq d$ and the $\alpha_{i,j}$'s are the polynomial's coefficients.

You are given two such polynomials $p(x, y)$ and $q(x, y)$, both of degree at most d. Give an algorithm that computes $p \cdot q$ in time[1] $O(d^2 \log d)$.

Hint: Reduce the problem to the univariate case by replacing y by an appropriate polynomial in x.

Exercise 4 (*10 points*)

a) (5 points) Given two sets A and B, where both sets contain integers between 0 and U, we want to compute a sumset $C = \{a + b : a \in A, b \in B\}$. Show that C can be computed in time $O(U \log U)$.

b) (5 points) You are now given a set A whose elements are all integers between 0 and U and a target integer t. We want to know if there exist elements a, b, c in A such that $a + b + c = t$ (this is known as the 3SUM problem). Give an $O(U \log U)$ time algorithm for this problem.

[1]More precisely, your algorithm may perform $O(d^2 \log d)$ operations in the ring R containing the coefficients.