Exercise 9: Recovering from Christmas

Task 1: Self-Stabilization Survey

a) Check out the algorithms from the lecture and the exercises. Make a table, marking each[1] of them as one of the following: (i) trivially self-stabilizing (this includes trivial modifications), (ii) well-suited for the transformation from the lecture, (iii) self-stabilizing with straightforward modifications, (iv) solving a problem unsuitable for a self-stabilizing algorithm, (v) you believe it can be made self-stabilizing, but it's not that simple, (vi) you believe it can't be made self-stabilizing, but it's not easy to show, and (vii) you haven't the foggiest idea.

b) Add a bit of explanation where you think it's helpful or needed.

c) Discuss unclear cases in the exercise session!

Task 2: Nice and Naughty

A class has students S. Some $N \subseteq S$ students are *naughty* and the others $S \setminus N$ are *nice*. If nice students get good grades for their exercise sheets, they stay nice; naughty students stay naughty while receiving good grades because they get away with it. If nice students receive bad grades, they turn naughty out of frustration. However, a naughty student $s \in N$ eventually sees reason and turns nice when consistently getting bad grades for $k_s \in \mathbb{N} \cup \{\infty\}$ times.

Since we have a perfect TA, he can correctly and deterministically decide whether a student was nice or naughty from the solutions that have (or have not) been handed in. Unfortunately, this takes two weeks, and he needs to determine grades already within one week. He thus must decide on the grades for week W based on niceness/naughtiness for weeks $1, \ldots, W-1$ only.

a) Devise a grading scheme that is self-stabilizing, where we consider an execution suffix correct iff all nice students remain nice and all naughty students remain naughty. What is the stabilization time?

b) We say that $s \in S$ has a *good heart* iff $k_s \neq \infty$. Improve the grading scheme such that it is still self-stabilizing if we keep the requirement that all nice students remain nice, and add the additional requirement that students with good hearts eventually become (and remain) nice, while the remaining students receive bad grades. What is the stabilization time?

c) Show that there is an algorithm $\mathcal{A}_k$ that can stabilize faster than that in b) if $k_s = k \neq \infty$ for all naughty students $s \in N$. (It should still stabilize if this is not the case, just be faster if it holds true!)

d) Argue that, in some sense, your solution from b) is doing well.

e)* Be nice!

Task 3*: Have a Happy New Vote

a) Program a simulator for Democrats vs. Republicans!

b) Make the simulator determine the number of steps until stabilization!

[1] There's no need to make separate entries for algorithms that are essentially the same, e.g., the various variants of Cole-Vishkin we encountered. (Unless this changes their category, of course.)

c) Come up with examples maximizing the time until stabilization as a function of the number of nodes n!

d) Show off your results in the exercise session!