Exercise 2: Flirting with Synchrony and Asynchrony

Task 1: Growing Balls

Denote by $B(v, r)$ the ball of radius r around v, i.e., $B(v, r) = \{u \in V : \text{dist}(u, v) \leq r\}$. Consider the following partitioning algorithm.

Algorithm 1 Cluster construction. $\rho \geq 2$ is a given parameter.

```
while there are unprocessed nodes do
    select an arbitrary unprocessed node v;
    r := 0;
    while |B(v, r + 1)| > ρ|B(v, r)| do
        r := r + 1
    end while
    makeCluster(B(v, r))                // all nodes in B(v, r) are now processed
    remove all cluster nodes from the current graph
end while
select intercluster edges
```

a) Show that Algorithm 1 constructs clusters of radius at most $\log_\rho n$.

b) Show that Algorithm 1 produces at most ρn intercluster edges.

c) For $k \in \{1, \ldots, \lceil \log n \rceil\}$, determine an appropriate choice $\rho(k)$, proving the precondition of Corollary 2.14!

Task 2: Showing Dijkstra, and Bellman & Ford the Ropes

a) Show that if the asynchronous Bellman-Ford algorithm from the lecture is executed synchronously, it sends only $\mathcal{O}(|E|)$ messages.

b) Use this to construct an asynchronous BFS tree construction algorithm of time complexity $\mathcal{O}(D)$ that uses $\mathcal{O}(|E|D)$ messages and terminates. You may assume that D is known here.

c) Can you give an asynchronous Bellmann-Ford-based algorithm that sends $\mathcal{O}(|E| + nD)$ messages and runs for $\mathcal{O}(D^2)$ rounds?

   Hint: Either answer is feasible, provided it is backed up by appropriate reasoning!

Task 3*: Liaison with Leslie Lamport

a) Look up what Lamport causality, Lamport clocks, and Lamport vector clocks are.

b) Contemplate their relation to synchronizers and what you've learned in the lecture.

c) Discuss your findings in the exercise session!