Multivariate Algorithmics:
Problem set 2

Dr. Karl Bringmann, Dr. Holger Dell, Cornelius Brand, Marc Roth
http://bit.ly/MulAlg18

Due: Before the lecture on November 06, 2018 Tutorial: November 08, 2018

Problem 2.1 First, read Section 2.6 about the Sunflower Lemma in the book, including its application to the d-hitting set problem. Then, using similar ideas, solve the following exercise on the d-set packing problem: You are given an integer k and a family of subsets $\mathcal{A}$ of a universe U, where each set in $\mathcal{A}$ is of size at most d. Decide whether there exist k sets $S_1, \ldots, S_k \in \mathcal{A}$ that are pairwise disjoint. Use the Sunflower Lemma to obtain a kernel for this problem with $f(d) \cdot k^d$ sets, for some computable f.

Problem 2.2 (✐) Fix an integer $r > 0$. Show that the clique and independent set problem, when parameterized by the solution size, are fixed-parameter tractable on r-regular graphs. Show that this remains true if r is not fixed anymore and we parameterize by $k + r$.

Problem 2.3 (✐) Prove: The vertex cover problem can be solved optimally in polynomial time on graphs of maximum degree at most 2.

Problem 2.4 Show that a graph on n vertices of minimum degree at least 3 contains a cycle of length at most $2\lceil \log n \rceil$. Use this to design a $(\log n)^{O(k)} \cdot n^{O(1)}$-time algorithm for the feedback vertex set problem on undirected graphs. Does this runtime bound suffice the conditions for an fpt-algorithm, i.e., can it be bounded by $f(k) \cdot n^{O(1)}$ for some computable f?

Problem 2.5 (☠) A graph is *chordal* if it does not contain a cycle on at least four vertices as an induced subgraph. That is, for every cycle of length at least four, there is at least one edge in the graph between to vertices of the cycle that is not in the cycle. Such an edge is a *chord*. Hence the name.

A *triangulation* of a graph $G = (V, E)$ is a set of edges $E' \subseteq \binom{V}{2}$ such that $(V, E \cup E')$ is chordal.

Consider the chordal completion problem: Given a graph and an integer k, decide whether G has a triangulation of size at most k.

a) Give a $k^{O(k)} n^{O(1)}$-time algorithm for the problem.

b) Show that there is a bijection between inclusion-wise minimal triangulations of a cycle of length ℓ and binary trees with $\ell - 2$ internal nodes. From this, conclude that a cycle on ℓ vertices has at most $4^{\ell-2}$ inclusion-wise minimal triangulations.

c) Use the previous point to design an algorithm for the problem running in time $2^{O(k)} \cdot n^{O(1)}$.

Problem 2.6 (☠) Consider the following problem: Given a connected graph G and an integer k, decide whether there is a spanning tree of G with at least k leaves. Design an algorithm that solves this problem in time $4^k \cdot n^{O(1)}$.

Please note:

- Each problem is worth one point. There are no half points.
- You need a third of all points from the problem sets to be admitted to the exam.
- You can hand in assignments in groups of size up to three.