Multivariate Algorithmics: Problem set 3

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`http://bit.ly/MulAlg18`

Due: Before the lecture on November 20, 2018 Tutorial: November 22, 2018

Problem 3.1 Obtain an algorithm for 3-*Hitting Set* running in time $2.4656^k n^{O(1)}$ using iterative compression. Generalize this algorithm to obtain an algorithm for d-*Hitting Set* running in time

$$((d-1)+0.4656)^k n^{O(1)} .$$

Problem 3.2 (☠) A set $X \subseteq V(G)$ of an undirected graph G is called an *independent feedback vertex set* if $G[X]$ is independent and $G - X$ is acyclic. In the *Independent Feedback Vertex Set* problem, we are given as input a graph G and a positive integer k, and the objective is to decide whether G has an independent feedback vertex set of size at most k. Show that this problem is fixed-parameter tractable by obtaining an algorithm running in time $5^k n^{O(1)}$ using iterative compression.

Problem 3.3 (🍃) Design a randomized polynomial-time algorithm $\mathbb{A}$ that, given a graph G and a positive integer k, outputs 0 with probability 1 if G has no vertex cover of size k and outputs 1 with probability $\geq 2^{-k}$ if G has a vertex cover of size k.

Then use $\mathbb{A}$ to obtain a randomized algorithm for Vertex Cover running in time $2^{O(k)} n^{O(1)}$ that succeeds in the positive case with probability $\geq 1/2$ and give a formal proof of the bound on the success probability.

In the subsequent exercises, a *randomized algorithm* is assumed to have one-sided error with constant success probability.

Problem 3.4 (🍃) In the *Triangle Packing* problem, we are given an undirected graph G and a positive integer k, and the objective is to test whether G has k vertex-disjoint triangles. Using color coding show that the problem admits a randomized algorithm with running time $2^{O(k)} n^{O(1)}$.

Problem 3.5 In the *Tree Subgraph Isomorphism* problem, we are given an undirected graph G and a tree T on k vertices, and the objective is to decide whether there exists a subgraph in G that is isomorphic to T. Obtain a $2^{O(k)} n^{O(1)}$-time randomized algorithm for the problem using color coding.

Problem 3.6 (☠) Consider the following problem: Given an undirected graph G and positive integers k and q, find a set X of at most k vertices such that $G - X$ has at least two components of size at least q.

a) Show that this problem can be solved in time $2^{O(q+k)} n^{O(1)}$ by a randomized algorithm.

b) Assuming $q > k$, show that the problem can be solved in time $q^{O(k)} n^{O(1)}$ by a randomized algorithm.

Please note:

- Each problem is worth one point. There are no half points.
- You need a third of all points from the problem sets to be admitted to the exam.
- You can hand in assignments in groups of size up to three.