If you have questions regarding the exercises, please ask them on the mailing list.
Please hand in your solutions by sending them to Ben at bwiederh@mpi-inf.mpg.de or directly in the lecture. Do *not* send them to the public mailing list.
The due date is listed on the website. Due time is the start of the next lecture.

Exercise 2: Flirting with Synchrony and Asynchrony

Task 1: Growing Balls

Denote by $B(v,r)$ the ball of radius r around v, i.e., $B(v,r) = \{u \in V : \text{dist}(u,v) \leq r\}$. Consider the following partitioning algorithm.

Algorithm 1 Cluster construction. $\rho \geq 2$ is a given parameter.

```
while there are unprocessed nodes do
    select an arbitrary unprocessed node v;
    r := 0;
    while |B(v, r + 1)| > ρ|B(v, r)| do
        r := r + 1
    end while
    makeCluster(B(v, r))          // all nodes in B(v, r) are now processed
    remove all cluster nodes from the current graph
end while
select intercluster edges
```

a) Show that Algorithm 1 constructs clusters of radius at most $\log_\rho n$.

b) Show that Algorithm 1 produces at most ρn intercluster edges.

c) For given $k \in \{1, \dots, \lceil \log n \rceil\}$, determine an appropriate choice $\rho(k)$, proving the precondition of Corollary 2.14!

   Hint: As a short-hand, we often don't write out common terms like n that are assumed to be globally known. Specifically, $\rho(k)$ may also depend on n, as if we had written $\rho(k,n)$. If in doubt, then we weren't clear enough, so tell us!

Task 2: Showing Dijkstra, and Bellman & Ford the Ropes

a) Show that if the asynchronous Bellman-Ford algorithm from the lecture is executed synchronously, it sends only $\mathcal{O}(|E|)$ messages.

b) Use this to construct an asynchronous BFS tree construction algorithm of time complexity $\mathcal{O}(D)$ that uses $\mathcal{O}(|E|D)$ messages and terminates. You may assume that D is known here.

c) Can you give an asynchronous Bellman-Ford-based algorithm that sends $\mathcal{O}(|E|+nD)$ messages and runs for $\mathcal{O}(D^2)$ rounds?

   Hint: Either answer is feasible, provided it is backed up by appropriate reasoning!

Task 3*: Liaison with Leslie Lamport

a) Look up what potential causality, Lamport clocks, and vector clocks are.

b) Contemplate their relation to synchronizers and what you've learned in the lecture.

c) Discuss your findings in the exercise session!