Topics in Algorithmic Game Theory and Economics

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Lecture 8
Some Mechanism Design

Mechanism Design

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We focus mostly on (online) auctions.

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A (deterministic) mechanism (x, p) for selling an item to one of n bidders is given by an allocation rule $x : \mathbb{R}^n \to \{0, 1\}^n$ with $\sum_i x_i \leq 1$, and pricing rule $p : \mathbb{R}^n \to \mathbb{R}$.

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- How should we design auction to prevent undesirable outcomes?

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$$\text{“Revenue for seller”} + \text{“Player utilities”} = \sum_i v_i x_i(b) = v_{i^*}$$

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Second price auction has many desirable properties.

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Mechanism (x, p) incentivizes truthful bidding if for every bidder i, alternative bid b_i', and bids $b_{-i} = (b_1, \ldots, b_{i-1}, b_{i+1}, b_n)$ of other bidders, it holds that

$$u_i(b_{-i}, v_i) \geq u_i(b_{-i}, b_i'),$$

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Bidders have non-negative utility (when reporting truthfully).

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$$u_i(b_{-i}, v_i) \geq u_i(b_{-i}, b_i'),$$

where $u_i(b) = x_i(b)(v_i - p(b))$.

Bidders have non-negative utility (when reporting truthfully).

Definition (Individually rational)

Mechanism (x, p) is individually rational if for every bidder i it holds

$$u_i(b) \geq 0$$

for every bid vector $b = (b_1, \ldots, b_{i-1}, v_i, b_{i+1}, \ldots, b_n)$.

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Definition (Welfare maximization)

Mechanism (x, p) is welfare maximizer if it maximizes

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Definition (Computational efficiency)

Mechanism (x, p) should be implementable in polynomial time, i.e., compute allocation x and price p in polynomial time.

Proof strategyproofness (second price auction):

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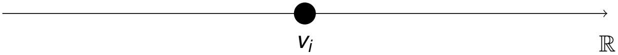

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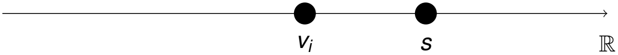

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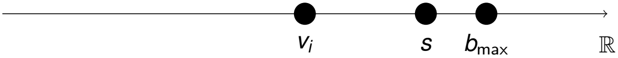

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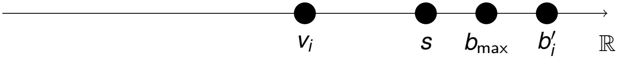

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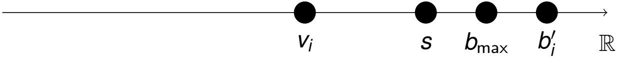

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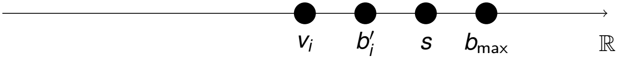

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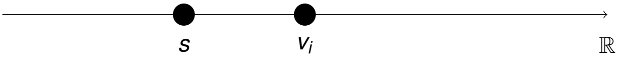

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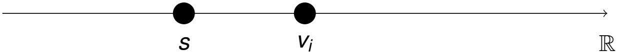

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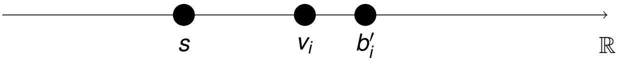

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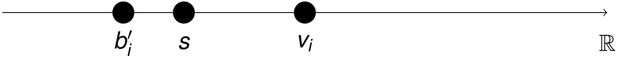

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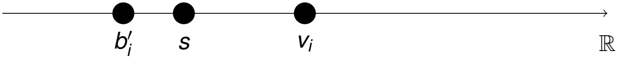

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Exercise: Show second price auction has monotone allocation rule.

Selling multiple items

Unit-demand setting

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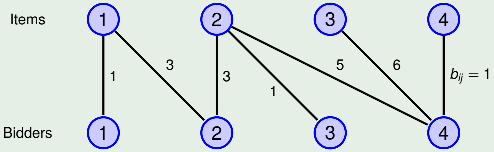
The goal is to assign (at most) one item to every bidder.

Example

Non-existing edges have $b_{ij} = 0$.



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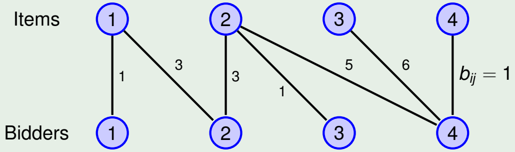

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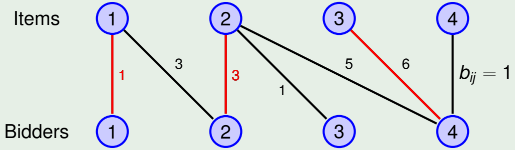

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$$x : \mathbb{R}_{\geq 0}^{n \times m} \to \{0, 1\}^{n \times m},$$

with $\sum_i x_{ij} \leq 1$ and $\sum_j x_{ij} \leq 1$, and pricing rule $p : \mathbb{R}_{\geq 0}^{n \times m} \to \mathbb{R}_{\geq 0}^{m}$.

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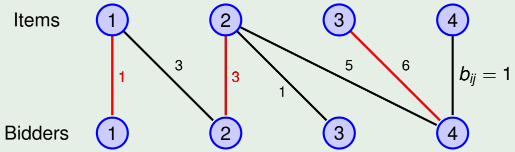

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- For bidder i, we have bid vector $b_i = (b_{i1}, \ldots, b_{im})$.

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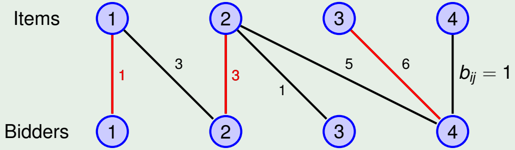

Definition (Mechanism)

A (deterministic) mechanism (x, p) is given by an allocation rule

$$x : \mathbb{R}_{\geq 0}^{n \times m} \to \{0, 1\}^{n \times m},$$

with $\sum_i x_{ij} \leq 1$ and $\sum_j x_{ij} \leq 1$, and pricing rule $p : \mathbb{R}_{\geq 0}^{n \times m} \to \mathbb{R}_{\geq 0}^{m}$.

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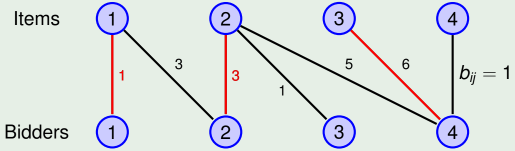

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OPT$(N \setminus \{i\}, M) -$ OPT$(N \setminus \{i\}, M \setminus \{j\})$ is welfare loss for other players by assigning j to i.

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Example

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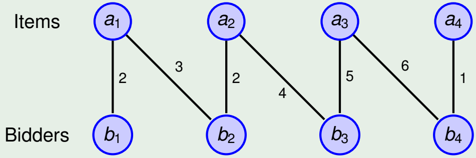

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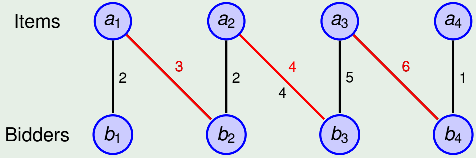

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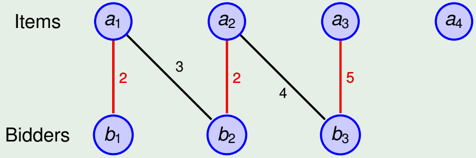

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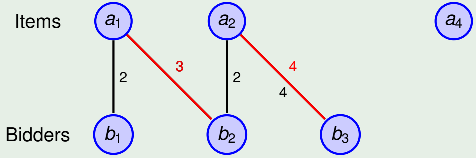

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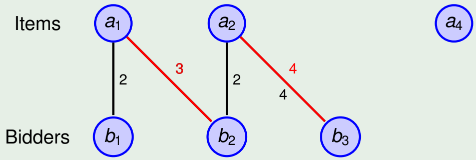

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- Price charged to bidder b_4 for item a_3 is

$$p_{43}(b) = 9 - 7 = 2.$$

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Online mechanism design

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Requirements for (online) deterministic mechanism (x, p):
Takes as input deterministic ordering $(y_1, \ldots, y_n)$ and bids $b_1, \ldots, b_n$ for the item.

- Specifies for every $k = 1, \ldots, n$ whether to allocate to y_k.
- This $\{0, 1\}$-variable x_k (and price p) for k is function of:
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- **Computationally tractable:** Decision on who to allocate item to, and computation of charged price, in poly-time.