Themistoklis Gouleakis Winter 2021

Property Testing, Exercise Sheet 1

https://www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/winter20/property-testing

Total Points: 40 Due: Thursday, **November 19**, 2020

You are allowed to collaborate on the exercise sheets, but you have to write down a solution on your own, ***using your own words****. Please indicate the names of your collaborators for each exercise you solve. Further, cite all external sources that you use (books, websites, research papers, etc.). You need to collect at least 50% of all points on exercise sheets to be admitted to the exam. Please send your solutions directly to Philip (*`wellnitz@mpi-inf.mpg.de`*).*

Exercise 1 — 10 points

Recall the proof of the lower bound on decision procedures for `MAJ` from the lecture. In particular, recall the following construction of the distribution X_n (Z_n): First select uniformly $i \in [n]$, and then select uniformly y from the binary strings of Hamming weight $\lfloor n/2 \rfloor$ that have a 0 at position i. Now set $X_n := y \oplus 0^{i-1}10^{n-i}$ (and $Z_n := y$).

Prove that X_n (and Z_n) is uniformly distributed over n-bit strings of Hamming weight $\lfloor n/2 \rfloor + 1$ (and $\lfloor n/2 \rfloor$).

Exercise 2 — 10 points

Show that the error probability of a property tester can be reduced to 2^{-t} at the cost of increasing its query (and time) complexity by a factor of $O(t)$, and while preserving one-sided error.

Exercise 3 — 10 points

Consider the following algorithm: On input $x \in \{0,1\}^n$, we select uniformly $i \in [n]$. If n is odd, we output x_i. If n is even and x_i is 1, we output 1 with probability $1 - n^{-1}$ and 0 otherwise.

Show that the above algorithm constitutes a two-sided error POT with linear detection probability for `MAJ`.

Exercise 4 — 5 + 5 points

Let `BAL` denote the set of "balanced" strings: `BAL` $:= \{x : \mathrm{wt}(x) = |x|/2\}$. Consider the following algorithm: On input $x \in \{0,1\}^n$, we select uniformly $i, j \in [n]$, and accept if and only if $x_i \neq x_j$.

a) Show that the above algorithm constitutes a two-sided error POT with threshold probability 0.5 and quadratic detection probability for `BAL`.

b) Generalize the above algorithm to the set S_c of "c-skewed" strings (for every $c \in (0, 0, 5)$), where

$$S_c := \{x : c \cdot |x| \leq wt(x) \leq (1-c) \cdot |x|\}.$$

Also compute the threshold and detection probabilities for the generalized algorithm.