$\leq$ on $\mathbb{N}$

$k \leq l$ iff

$\exists j \; k + j = l$

for $k, l, j \in \mathbb{N}$

$<$

$<$ on $\mathbb{N}$

wellfounded on $\mathbb{N}$

but not on $\mathbb{Z}$

$\subset$ on sets of nats.

$S \subset T$ iff

$\exists D \neq \emptyset$ s.t. $S \cup D = T$

$\{1\}$ $\{2, 3\}$

$\{1,1,1\} \not> \{1,1\}$

$\{1,1,2\} \cup \{1,3,3\} = \{1,1,1,2,3,3\}$

$\text{— " —} \cap \text{— " —} = \{1\}$

$\{2\} > \{1 \ldots \ldots 1\}$

$\{2,1,1\} > \{2\}$

$\{2,1,1\} = \{1,1,2\}$

$b > a$

$b > ab > aab > aaab > \ldots$

$P(n)$ $\quad 0 + 1 + \ldots + n = \frac{n(n+1)}{2}$

Base case:

$P(0): \quad 0 = \frac{0(0+1)}{2} = 0$

Induction step:

For every $k > 0$ $P(k)$ holds if

$P(k')$ holds for all $k' < k$

Hypothesis:

$P(k')$ holds for all $k' < k$

$\Rightarrow P(k-1)$ holds

$$0 + 1 + \ldots + n - 1 + n = \frac{(n-1)n}{2} + n$$

$$= \frac{(n+1) \cdot n}{2}$$

M = set of finite sequences from $\{a, b, c\}$

$$R = \left\{ \begin{array}{l} xbay \to xaby, \\ xcby \to xbcy, \\ xcay \to xacy \end{array} \right\} \quad x, y \in M$$

$$cbacba \to cabcba \to acbcba$$

$$\to^* aabbcc$$

$$ccbbaa$$

$$R = \{a \to b, a \to c, c \to a, b \to a\}$$

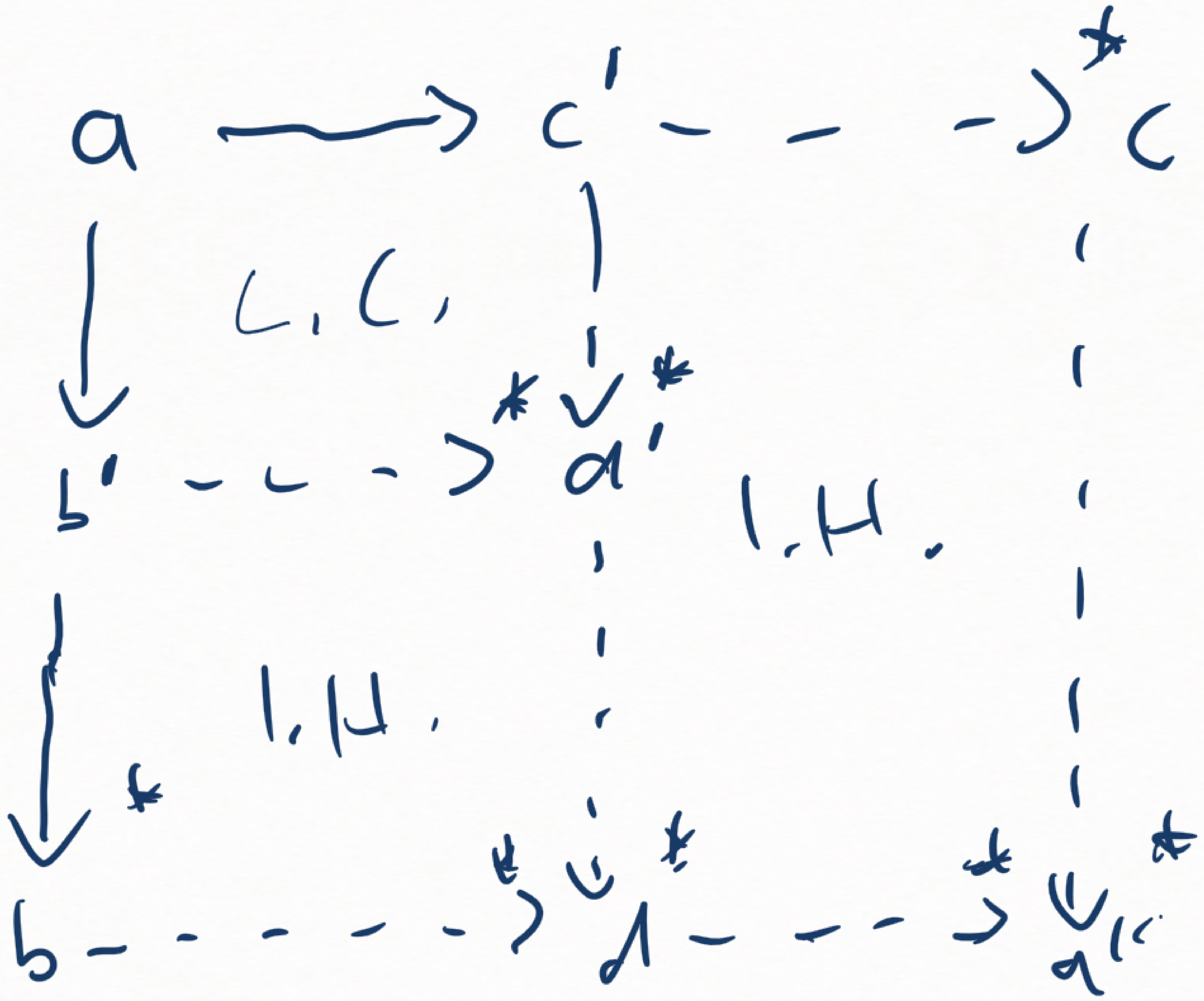
$$c \downarrow b$$

$$a_1 \leftrightarrow^* a_n \qquad a_1 \leftrightarrow a_2 \leftrightarrow \dots \leftrightarrow a_n$$

$$a_{i-1} \leftarrow a_i \to a_{i+1}$$

$R = \{ b \to a,\ a \to b,\ b \to c \}$

$\to$ is terminating $\Rightarrow$ $\to$ is a well-founded ordering.
So we prove $Q(a) =$ "a is confluent" by induction.



$\{3x + 4y = 4,\ x - y = 6\}$

$y = 1 - \frac{3}{4}x \qquad y = x - 6$

$\frac{7}{4}x = 7$

$y = z + 1 \qquad z = x + 1 \qquad x = y + 1$

$y = x + 2 \qquad z = y + 2 \qquad x = z + 2$

$y = y + 4 \qquad z = y + 4 \qquad x = y + 4$

~~x~~ $4 = 2x \qquad 2 = x$