Factoring

$R(a,b) \vee P(f(a))$ $\quad R(a, f(a)) \vee P(f(a)) \vee P(f(a))$

$$\xRightarrow[\text{Fac}]{} R(a, f(a)) \vee P(f(a))$$

does not subsume

$$R(x, z) \vee P(z) \vee P(f(x))$$

$$\{x \to a, z \to f(a)\}$$

$$R(x, f(x)) \vee P(f(x))$$

N model iff N has a Herbrand model

countably infinite

$P(a)^*$

$\neg P(x) \vee P(g(x))^*$

$g > a$

$H = \{P(a), P(g(a)), P(g(g(a))), \vdots\}$

$= N_I$

Completeness: 3.12.9

by contradiction $\bot \notin N$,

N sat, $N_I \not\models N$

there is a smallest clause $C \in N$ and $N_I \not\models C$

(i) $C = C' \vee P(t_1, \dots, t_n)$ and $P(t_1, \dots, t_n)$ is max lit

nothing selected

if $P(t_1, \dots, t_n)$ not strict max $\rightsquigarrow P(t_1, \dots, t_n) \in C'$

$\rightsquigarrow$ Apply Factoring $\rightsquigarrow C' < C$, $N_I \not\models C'$ ↯

↯ C was minimal ↯

(ii) $C = C' \vee \neg P(t_1, \dots, t_n)$ and $\neg P(t_1, \dots, t_n)$ max or selected

$N_I \not\models C \rightsquigarrow P(t_1, \dots, t_n) \in N_I \rightsquigarrow D = D' \vee P(t_1, \dots, t_n)$ ← strict max

$$\frac{C' \vee \neg P(t_1,\dots,t_n) \succ \underline{D' \vee P(t_1,\dots,t_n)}}{\neg P(t_1,\dots,t_n) \succ D' \vee \underbrace{P(t_1,\dots,t_n)}_{\text{strictly max}}}$$

$\overset{\text{similarly}}{\Longrightarrow}$

$$\frac{C' \vee D'}{N_I \nvDash D'} \quad \text{otherwise no production}$$

$N_I \nvDash C'$

$N_I \nvDash C' \vee D'$

$C' \vee D' \prec C' \vee \neg P(t_1,\dots,t_n)$

$\leadsto$ ↯ $C' \vee \neg P(t_1,\dots,t_n)$ was the minimal false clause

$P(f(a))^* \vee R(a,b)$

Res

$P(f(a))^+ \vee R(f(a),b)^*$ KBO $w(*) = 1$

Res

$P(f(a))^* \vee P(a) \vee P(a)$ $f > P > R > a > b$

Fac

Superposition: saturated

select $\neg P f(a))$

with selection

$R(a,b) \vee R(f(a),b)^*$

$R(f(a,b))^* \vee P(a) \vee P(a)$

$$\neg P(a)^{+} \vee R(f(a,b))^{*}$$

$$\neg R(f(a,b))^{*} \vee \neg R(f(b,a))^{+} \vee R(f(a,a))$$

$$f > P > R > a > b$$

3.12.10 Compactness

N (countable infinite) but unsatisfiable

$\longrightarrow$ saturation N^*

Superposition

$\bot \in N^*$

look how $\bot$ was derived

$\leadsto$ by finitely many clauses $N' \subseteq N$

$\leadsto$ N' unsat

N' finite

Ground

$$C \vee P(t_1, \ldots, t_n) \qquad D \vee \neg P(t_1, \ldots, t_n)$$

$P(t \ldots)$ strict max, for all $L \in C$: $A < P(t_1, \ldots, t_n)$

$\neg P(-)$ max, for all $L \in D$: $D \leq \neg P(t_1, \ldots, t_n)$

First-Order

$$C \vee P(f(x,y)) \qquad D \vee \neg P(f(x',y'))$$

$P(f(x,y))$ strict max $\rightsquigarrow$ no $L \in C$ s.t. $L \geq P(f(x,y))$

$\neg P(f(x',y'))$ max $\rightsquigarrow$ no $L \in D$ s.t. $L > \neg P(f(x',y'))$

Superposition with Variables

$$a \succ b \succ R \qquad \left(\begin{array}{l} \neg R(x,y)^{*} \vee R(y,x)^{*} \\ R(b,a)^{*} \end{array} \right. \qquad \text{mgu}_E\{R(x,y) = R(b,a)\}$$

$$\rightsquigarrow \quad R(a,b) \qquad \sigma = \{x \to b, y \to a\}$$

$\neg R(b,a)$ is strictly max in $\neg R(b,a) \vee R(a,b)^{*}$