

$$g(g(g(a))) \approx a \wedge g(g(g(g(g(a)))) \approx a \wedge g(a) \neq a$$

$$\begin{aligned} \Rightarrow & \text{Flattening} \\ & \text{CC} \quad g(a) \approx c_1 \wedge g(g(c_1)) \approx a \wedge \\ & \quad g(g(g(g(c_1)))) \approx a \end{aligned}$$

$$\begin{aligned} \Rightarrow & \text{Flattening}^* \\ & \text{CC} \quad g(a) \approx c_1 \wedge g(c_1) \approx c_2 \wedge g(c_2) \approx c_3 \\ & \quad \wedge g(c_3) \approx c_4 \wedge c_3 \approx a \wedge g(c_4) \approx a \\ & \quad a < c_1 < c_2 < c_3 < c_4 \end{aligned}$$

$$E_0 = \{c_3 \approx a\} \quad R_0 = \{g(a) \rightarrow c_1, g(c_1) \rightarrow c_2, g(c_2) \rightarrow c_3, \\ g(c_3) \rightarrow c_4, g(c_4) \rightarrow a\}$$

$$(E_0, R_0) \Rightarrow_{\text{orient CC}} (\emptyset, \{c_3 \rightarrow a\} \cup R_0)$$

$$\Rightarrow_{\text{collapse CC}} (\emptyset, \{g(a) \rightarrow c_1, g(c_1) \rightarrow c_2, g(c_2) \rightarrow c_3, \\ g(a) \rightarrow c_4, g(c_4) \rightarrow a, c_3 \rightarrow a\})$$

\Rightarrow Deduce \mathcal{C} $(\{ c_4 \approx c_1 \}, \{ g(a) \rightarrow c_1, g(c_1) \rightarrow c_2, g(c_2) \rightarrow c_3, g(c_4) \rightarrow a, c_3 \rightarrow a \})$

\Rightarrow Orient \mathcal{C} $(\emptyset, \{ c_4 \rightarrow c_1 \} \cup \mathcal{R}_1)$

\Rightarrow Collapse \mathcal{C} $(\emptyset, \{ g(a) \rightarrow c_1, g(c_1) \rightarrow c_2, g(c_2) \rightarrow c_3, g(c_1) \rightarrow a, c_3 \rightarrow a, c_4 \rightarrow c_1 \})$

\Rightarrow Deduce \mathcal{C} $(\{ c_2 \approx a \}, \{ g(a) \rightarrow c_1, g(c_2) \rightarrow c_3, g(c_1) \rightarrow a, c_3 \rightarrow a, c_4 \rightarrow c_1 \})$

\Rightarrow Orient \mathcal{C} $(\emptyset, \{ c_2 \rightarrow a \} \cup \mathcal{R}_2)$

\Rightarrow Collapse \mathcal{C} $(\emptyset, \{ g(a) \rightarrow c_1, g(a) \rightarrow c_3, g(c_1) \rightarrow a, c_3 \rightarrow a, c_4 \rightarrow c_1, c_2 \rightarrow a \})$

\Rightarrow Deduce \mathcal{C} $(\{ c_1 \approx c_3 \}, \{ g(a) \rightarrow c_1, g(c_1) \rightarrow a, c_3 \rightarrow a, c_4 \rightarrow c_1, c_2 \rightarrow a \})$

\Rightarrow Simplify \mathcal{C} $(\{ c_1 \approx a \}, \{ c_3 \rightarrow a, c_4 \rightarrow c_1, c_2 \rightarrow a \})$

$$\Rightarrow (\{a \approx c_1\}, \{g(u) \rightarrow c_1, g(c_1) \rightarrow u, c_3 \rightarrow a, c_4 \rightarrow u, c_2 \rightarrow a\})$$

$$\Rightarrow \text{Orinut } c(\emptyset, \{c_1 \rightarrow a\} \cup \{c_4\})$$

$$\Rightarrow \text{Co-Union } c(\emptyset, \{g(u) \rightarrow c_1, g(a) \rightarrow a, c_3 \rightarrow a, c_4 \rightarrow a, c_2 \rightarrow a, c_1 \rightarrow a\})$$

$$\Rightarrow \text{Deduce } c(\{c_1 \approx a\}, \{g(u) \rightarrow a, c_3 \rightarrow a, c_4 \rightarrow a, c_2 \rightarrow a, c_1 \rightarrow u\})$$

$$\Rightarrow \text{Simplify } c(\{a \approx a\}, \text{---} \parallel \text{---})$$

$$\Rightarrow \text{Delete } c(\emptyset, \{g(u) \rightarrow a, c_3 \rightarrow a, c_4 \rightarrow a, c_2 \rightarrow a, c_1 \rightarrow a\})$$

$$s \downarrow_R = t \downarrow_R \text{ iff } \exists k s \approx t \text{ in } R$$

$$g(u) \neq a \quad g(u) \downarrow_R \approx a \quad a \downarrow_R \approx a$$

$$f(x_1, 0) \geq x_3 \wedge f(x_1, 0) \leq x_3$$

after purification

$$x_5 \approx 0 \wedge f(x_1, x_5) \approx x_4 \wedge f(x_1, x_5) \approx x_6 \wedge x_4 \geq x_3 \wedge x_6 \leq x_3$$

$$\underbrace{(\{x_4 \geq x_3, x_6 \leq x_3, x_5 \approx 0\}, \emptyset)}_{N_1}, \underbrace{(\{f(x_1, x_5) \approx x_4, f(x_1, x_5) \approx x_6\}, \emptyset, \perp)}_{N_2}$$

$$\Rightarrow \text{Solve } (N_1, \emptyset, N_2, \{x_4 \approx x_6\}, \emptyset, \perp)$$

NO

$$\Rightarrow \text{Solve } (N_1 \cup \{x_4 \approx x_6\}, \{x_4 \approx x_3, x_6 \approx x_3\}, N_2 \cup \{x_4 \approx x_6\}, \emptyset, \perp)$$

NO

$$\Rightarrow \text{Success } (N_1 \cup \{x_4 \approx x_6\} \cup \{x_4 \approx x_3, x_6 \approx x_3\}, \emptyset, N_2 \cup \{x_4 \approx x_6\} \cup \{x_4 \approx x_3, x_6 \approx x_3\}, \emptyset, \perp)$$

NO