OCDCL

$N = \{ A \vee B, A \vee \neg B \}$

$cost(A) = cost(B) = 1$

$cost(\neg A) = cost(\neg B) = 0$

$M = \{ AB, A\neg B \}$ models of N

$cost(AB) = 1 + 1 = 2$

$cost(A\neg B) = 1 + 0 = 1$

$\Rightarrow$ cost optimal model: $A \neg B$

$$N = \{ A \vee B ,\ A \vee \neg B \} \qquad \mathit{cost}(A) = \mathit{cost}(B) = 1$$

$$\mathit{cost}(\neg A) = \mathit{cost}(\neg B) = 0$$

$$(\varepsilon; N; \emptyset; 0; \top; \varepsilon)$$

$$\xRightarrow[\text{ocdcl}]{\text{Decide}} (A^1; N; \emptyset; 1; \top; \varepsilon)$$

$$\xRightarrow[\text{ocdcl}]{\text{Decide}} (A^1 B^2; N; \emptyset; 2; \top; \varepsilon)$$

$$\xRightarrow[\text{ocdcl}]{\text{Improve}} (A^1 B^2; N; \emptyset; 2; \top; AB)$$

$$\xRightarrow[\text{ocdcl}]{\text{ConflOpt}} (A^1 B^2; N; \emptyset; 2; \neg A \vee \neg B; AB)$$

$$\xRightarrow[\text{ocdcl}]{\text{Backtrack}} (A^1 \neg B^{\neg A \vee \neg B}; N; \{\neg A \vee \neg B\}; 1; \top; AB)$$

$$\xRightarrow[\text{ocdcl}]{\text{Improve}} (A^1 \neg B^{\neg A \vee \neg B}; N; \{\neg A \vee \neg B\}; 1; \top; A\neg B)$$

$$\xRightarrow[\text{ocdcl}]{\text{ConflOpt}} (A^1 \neg B^{\neg A \vee \neg B}; N; \{\neg A \vee \neg B\}; 1; \neg A \vee B; A\neg B)$$

$$\xRightarrow[\text{ocdcl}]{\text{Resolve}} (A^1; N; \{\neg A \vee \neg B\}; 1; \neg A; A\neg B)$$

$$\xRightarrow[\text{ocdcl}]{\text{Backtrack}} (\neg A^{\neg A}; N; \{\neg A \vee \neg B, \neg A\}; 0; \top; A\neg B)$$

$$\xRightarrow[\text{ocdcl}]{\text{Propagate}} (\neg A^{\neg A} B^{A \vee B}; N; \{\neg A \vee \neg B, \neg A\}; 0; \top; A\neg B)$$

$$\xRightarrow[\text{ocdcl}]{\text{ConflSat}} (\neg A^{\neg A} B^{A \vee B}; N; \{\neg A \vee \neg B, \neg A\}; 0; A \vee \neg B; A\neg B)$$

$$\xRightarrow[\text{ocdcl}]{\text{Resolve}} (\neg A^{\neg A}; N; \{\neg A \vee \neg B, \neg A\}; 0; A; A\neg B)$$

$$\models_{oracle}^{Resolve} (\varepsilon; N; \{\neg A \vee \neg B, \neg A\}; 0; \bot; A \neg B)$$

final state with cost optimal model $A \neg B$

$$N = \{ A \vee B , A \vee \neg B \} \qquad cost(A) = cost(\neg A) = 1$$

$$cost(B) = cost(\neg B) = 0$$

$$(\varepsilon; N; \emptyset; 0; \top; \varepsilon)$$

$$\Rightarrow^{Decide}_{OCDCL} (A^1; N; \emptyset; 1; \top; \varepsilon)$$

$$\Rightarrow^{Decide}_{OCDCL} (A^1 B^2; N; \emptyset; 2; \top; \varepsilon)$$

$$\Rightarrow^{Improve}_{OCDCL} (A^1 B^2; N; \emptyset; 2; \top; AB)$$

$$\Rightarrow^{ConflOpt}_{OCDCL} (A^1 B^2; N; \emptyset; 2; \neg A \vee \neg B; AB)$$

$$\Rightarrow^{Backtrack}_{OCDCL} (A^1 \neg B^{\neg A \vee \neg B}; N; \{\neg A \vee \neg B\}; 1; \top; AB)$$

$$\Rightarrow^{ConflOpt}_{OCDCL} (A^1 \neg B^{\neg A \vee \neg B}; N; \{\neg A \vee \neg B\}; 1; \neg A \vee B; AB)$$

$$\Rightarrow^{Resolve}_{OCDCL} (A^1; N; \{\neg A \vee \neg B\}; 1; \neg A; AB)$$

$$\Rightarrow^{Backtrack}_{OCDCL} (\neg A^{\neg A}; N; \{\neg A \vee \neg B, \neg A\}; 0; \top; AB)$$

$$\Rightarrow^{Prune}_{OCDCL} (\neg A^{\neg A}; N; \{\neg A \vee \neg B, \neg A\}; 0; A; AB)$$

$$\Rightarrow^{Resolve}_{OCDCL} (\varepsilon; N; \{\neg A \vee \neg B, \neg A\}; 0; \bot; AB)$$

Max-SAT

$N_H = \{A \vee B\}$ $\quad N_S = \{\underbrace{\neg A \vee C}_{C_1}, \underbrace{B \vee C}_{C_2}\}$

$\omega(\neg A \vee C) = 1$

$\omega(B \vee C) = 2$

$cost(A B C) = cost(A \neg B C) = 0$

$cost(\neg A B C) = cost(\neg A B \neg C) = 0$

$cost(A B \neg C) = 1$ $\quad A B \neg C$ violates C_1

$cost(A \neg B \neg C) = 3$ $\quad A \neg B \neg C$ violates C_1, C_2

$N_S' = \{S_1 \vee \neg A \vee C, S_2 \vee B \vee C\}$

$N' = \{A \vee B, S_1 \vee \neg A \vee C, S_2 \vee B \vee C\}$

$cost(S_1) = 1$

$cost(S_2) = 2$

$cost(A) = cost(\neg A) = cost(B) = cost(\neg B) = 0$

$cost(C) = cost(\neg C) = cost(\neg S_1) = cost(\neg S_2) = 0$

$m_1 = A B C \neg S_1 \neg S_2$ and $cost(m_1) = 0$

$m_2 = A B \neg C S_1 \neg S_2$ and $cost(m_2) = 1$

Minimal Covering Models

$N = \{ A \vee B, \neg A \vee C, B \vee C \}$

$\mathcal{M} = \{ ABC, A\neg BC, \neg ABC, \neg AB\neg C \}$

$\mathcal{M}_1' = \{ ABC \}$ → minimal

$\mathcal{M}_2' = \{ A\neg BC, \neg ABC \}$

$\mathcal{M}_3' = \{ A\neg BC, \neg AB\neg C \}$

$\vdots$

$$N = \{ \underbrace{A \vee B}_{C1}, \underbrace{\neg A \vee C}_{C2}, \underbrace{B \vee C}_{C3} \}$$

$$N_1 := \{ A^1 \vee B^1 \vee \neg G_1, \neg A^1 \vee C^1 \vee \neg G_1, B^1 \vee C^1 \vee \neg G_1 \}$$

$$N_2 := \{ A^2 \vee B^2 \vee \neg G_2, \neg A^2 \vee C^2 \vee \neg G_2, B^2 \vee C^2 \vee \neg G_2 \}$$

$$N_3 := \{ A^3 \vee B^3 \vee \neg G_3, \neg A^3 \vee C^3 \vee \neg G_2, B^3 \vee C^2 \vee \neg G_3 \}$$

$$N_+ := \{ A^1 \vee A^2 \vee A^3, B^1 \vee B^2 \vee B^3, C^1 \vee C^2 \vee C^3 \}$$

$$\begin{aligned} N_G := \{ & \neg A^1 \vee G_1, \neg B^1 \vee G_1, \neg C^1 \vee G_1, \\ & \neg A^2 \vee G_2, \neg B^2 \vee G_2, \neg C^2 \vee G_2, \\ & \neg A^3 \vee G_3, \neg B^3 \vee G_3, \neg C^3 \vee G_3 \} \end{aligned}$$

$$N' = N_1 \cup N_2 \cup N_3 \cup N_+ \cup N_G$$

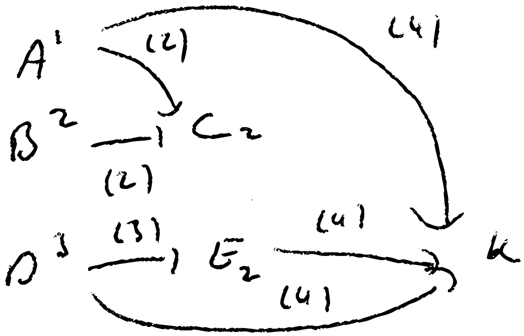
$$m_1 = A^1 G_1 B^1 C^1 \neg G_2 \neg A^2 \neg B^2 \neg C^2 \neg G_3 \neg A^3 \neg B^3 \neg C^3$$

$$\Rightarrow \mathrm{cnl}(m_1) = 1$$

CDCL Chrono

$N = \{ \underset{(1)}{A \vee B \vee C},\ \underset{(2)}{\neg A \vee \neg B \vee C},\ \underset{(3)}{\neg D \vee E},\ \underset{(4)}{\neg D \vee \neg A \vee \neg E},\ \underset{(5)}{D \vee F},\ \underset{(6)}{D \vee \neg F} \}$

$A^1\ B^2\ C_2^{(2)}\ D^3\ E_3^{(3)}$ ← level



$$\frac{\neg D \vee \neg A \vee \neg E \qquad \neg D \vee E}{\neg D \vee \neg A \quad (7)}$$

$\delta(D) = 3$ conflict level

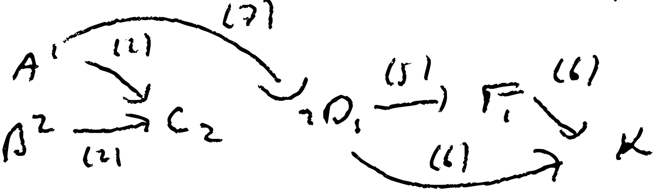
$\delta(A) = 1$ assertion level

↓

backtrack to level 2

propagate $\neg D$ at level 1

$A^1\ B^2\ C_2^{(2)}\ \neg D_1^{(7)}\ F_1^{(5)}$



$$\frac{D \vee \neg F \qquad D \vee F}{D}$$

$\delta(F) = 1$ conflict level

↓

backtrack to level 0

propagate D at level 0

implementation:

either

- backtrack to the conflict level before executing conflict analysis

or

- execute conflict analysis right away

$$N = \{ \underset{(1)}{A \lor B \lor C},\ \underset{(2)}{\neg A \lor \neg B \lor C},\ \underset{(3)}{\neg D \lor E},\ \underset{(4)}{\neg D \lor \neg A \lor \neg E},\ \underset{(5)}{D \lor F},\ \underset{(6)}{D \lor \neg F} \}$$

$$(\varepsilon; N; \emptyset; \delta_\infty; \top)$$

$$\Rightarrow^{\text{Decide}}_{\text{CDCLChrono}} (A^1; N; \emptyset; \delta[A \mapsto 1]; \top)$$

$$\Rightarrow^{\text{Decide}}_{\text{CDCLChrono}} (A^1 B^2; N; \emptyset; \delta[B \mapsto 2]; \top)$$

$$\Rightarrow^{\text{Propagate}}_{\text{CDCLChrono}} (A^1 B^2 C_2^{(2)}; N; \emptyset; \delta[C \mapsto 2]; \top)$$

$$\Rightarrow^{\text{Decide}}_{\text{CDCLChrono}} (A^1 B^2 C_2^{(2)} D^3; N; \emptyset; \delta[D \mapsto 3]; \top)$$

$$\Rightarrow^{\text{Propagate}}_{\text{CDCLChrono}} (A^1 B^2 C_2^{(2)} D^3 E_3^{(3)}; N; \emptyset; \delta[E \mapsto 3]; \top)$$

$$\Rightarrow^{\text{Conflict}}_{\text{CDCLChrono}} (A^1 B^2 C_2^{(2)} D^3 E_3^{(3)}; N; \emptyset; \delta; \neg D \lor \neg A \lor \neg E)$$

$$\Rightarrow^{\text{Resolve}}_{\text{CDCLChrono}} (A^1 B^2 C_2^{(2)} D^3; N; \emptyset; \delta[E \mapsto \infty]; \underbrace{\neg D \lor \neg A}_{(7)})$$

$$\Rightarrow^{\text{Backtrack}}_{\text{CDCLChrono}} (A^1 B^2 C_2^{(2)} \neg D_1^{(7)}; N; \{\neg D \lor \neg A\}; \delta[D \mapsto \infty][D \mapsto 1]; \top)$$

$$\Rightarrow^{\text{Propagate}}_{\text{CDCLChrono}} (A^1 B^2 C_2^{(2)} \neg D_1^{(7)} F_1^{(5)}; N; \{\neg D \lor \neg A\}; \delta[F \mapsto 1]; \top)$$

$$\Rightarrow^{\text{Conflict}}_{\text{CDCLChrono}} (A^1 B^2 C_2^{(2)} \neg D_1^{(7)} F_1^{(5)}; N; \{\neg D \lor \neg A\}; \delta; D \lor \neg F)$$

$$\Rightarrow_{\text{Resolve}}^{\text{CDCLChrono}} (A^1 B^2 C_2^{(2)} \neg D_1^{(7)}; N; \{\neg D \vee \neg A\}; \delta[F \mapsto \infty]; \underbrace{D}_{(8)})$$

$$\Rightarrow_{\text{Backtrack}}^{\text{CDCLChrono}} (D_0; N; \{\neg D \vee \neg A, D\}; \delta[D \mapsto \infty][C \mapsto \infty][B \mapsto \infty][A \mapsto \infty][D \mapsto 0]; \top)$$

$$\Rightarrow_{\text{Propagate}}^{\text{CDCLChrono}} (D_0^{(8)} E_0^{(3)}; \{(7), (8)\}; \delta[E \mapsto 0]; \top)$$

$$\Rightarrow_{\text{Propagate}}^{\text{CDCLChrono}} (D_0^{(8)} E_0^{(3)} \neg A_0^{(4)}; N; \{(7), (8)\}; \delta[\neg A \mapsto 0]; \top)$$

$$\Rightarrow_{\text{Decide}}^{\text{CDCLChrono}} (D_0^{(8)} E_0^{(3)} \neg A_0^{(4)} B^1; N; \{(7), (8)\}; \delta[B \mapsto 1]; \top)$$

$$\Rightarrow_{\text{Decide}}^{\text{CDCLChrono}} (D_0^{(8)} E_0^{(3)} \neg A_0^{(4)} B^1 C^1; N; \{(7), (8)\}; \delta[C \mapsto 2]; \top)$$

Terminal state with model $\neg ABCDE$

$\Rightarrow$ satisfiable