$$\varphi = (P \to Q) \wedge (P \to R) \quad P, Q, R \in \Sigma$$

$$\mathcal{A}(P) = 1, \quad \mathcal{A}(Q) = 1, \quad \mathcal{A}(R) = 1$$

$$\begin{aligned}
\mathcal{A}(\varphi) &= \min(\{\underline{\mathcal{A}(P \to Q)}, \underline{\mathcal{A}(P \to R)}\}) \\
&= \min(\{\max(1 - \mathcal{A}(P), \mathcal{A}(Q)\}, \\
&\qquad \{\max(1 - \mathcal{A}(P), \mathcal{A}(R)\}) \\
&= \min(\{1, 1\}) = \underline{1} \qquad \mathcal{A} \models \varphi
\end{aligned}$$

$$\varphi = (P \to Q) \wedge (P \to R)$$

$$\mathcal{A}(P) = 1, \quad \mathcal{A}(Q) = \mathcal{A}(R) = 0$$

$$\mathcal{A}(\varphi) = 0 \Rightarrow \mathcal{A} \not\models \varphi$$

$$\mathcal{A}'(P) = 0$$

$0 \to 0$ ✓

$0 \to 1$ ✓

$1 \to 1$ ✓

$1 \to 0$ ✗

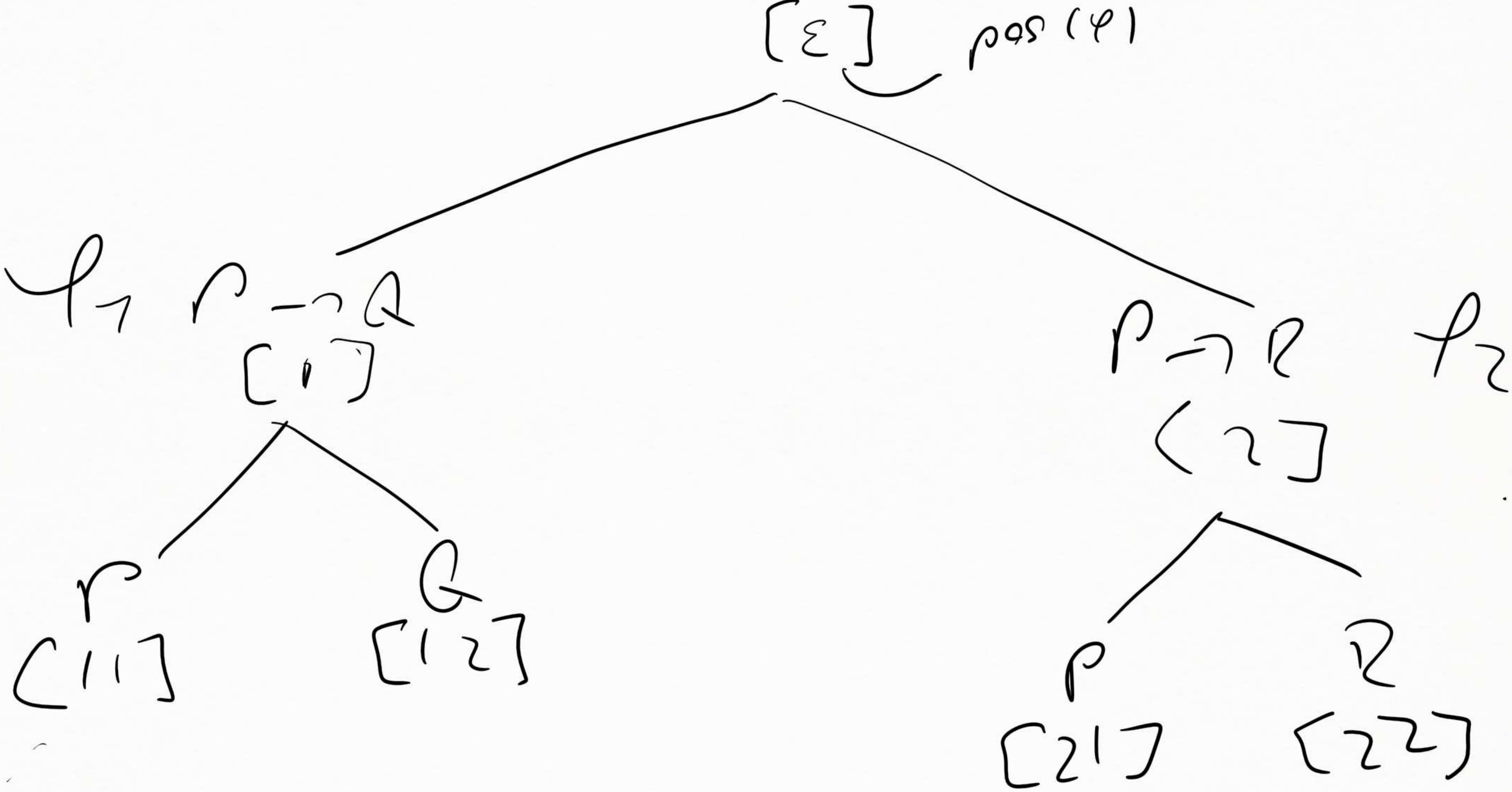
$$\varphi = ((P \vee Q) \wedge (P \vee \neg Q)) \wedge ((\neg P \vee Q) \wedge (\neg P \vee \neg Q))$$

φ valid iff $\neg\varphi$ unsatisfiable

$P \vee \neg P$ $\quad$ $\varphi \vee \neg \varphi$

$\varphi \leftrightarrow \psi$ iff $(\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$

$$\varphi = \overbrace{(P \to Q)} \wedge \overbrace{(P \to R)}$$



$$\varphi = \underbrace{(P \to Q)}_{\varphi_1} \wedge \underbrace{(P \to R)}_{\varphi_2}$$

$$pos(\varphi) = \{\varepsilon\} \cup \{1p \mid p \in pos(\varphi_1)\} \cup \{2p \mid p \in pos(\varphi_2)\}$$

$$pos(\varphi_1) = \{\varepsilon\} \cup \{1p \mid p \in pos(P)\} \cup \{2p \mid p \in pos(Q)\}$$

$$pos(\varphi_2) = \{\varepsilon\} \cup \{1p \mid p \in pos(P)\} \cup \{2p \mid p \in pos(R)\}$$

$$pos(\varphi_1) = \{\varepsilon, 1, 2\}$$

$$pos(\varphi_2) = \{\varepsilon, 1, 2\}$$

$$pos(\varphi) = \{\varepsilon, 1, 11, 12, 2, 21, 22\}$$

$$|pos(\varphi)| = 7 \quad \text{size of } \varphi$$

$$\varphi = \underbrace{(\overbrace{P}^{11} \to \overbrace{Q}^{12})}_{1} \wedge \underbrace{(\overbrace{P}^{21} \to R)}_{2}$$

$$\varphi|_{\varepsilon} = \varphi$$

$$\varphi|_{1} = P \to Q \qquad \varphi|_{21} = P$$

$$\varphi|_{12} = Q$$

$$\varphi[Q]_{21} = (P \to Q) \wedge (Q \to R)$$

$$\varphi[Q \to R]_{1} = (Q \to R) \wedge (P \to R)$$

$P\ Q\ \neg R$ $\quad \{P, Q, \neg R\}$ Σ-valuation

$P\ R\ Q$ $\quad \{P, Q\}$ Herbrand interpretation

Deduction Theorem: $\varphi \models \psi$ iff $\models \varphi \to \psi$

Proof. ($\Rightarrow$) Suppose $\varphi \models \psi$. Thee exists an

arbitrary Σ-deduction $\mathcal{A}$

Supp. $\mathcal{A}(\varphi) = 1$, then $\mathcal{A}(\varphi \to \psi) =$

$\Downarrow$

$\max(\{\underbrace{1 - \mathcal{A}(\varphi)}_{0}, \underbrace{\mathcal{A}(\psi)}_{1}\}) = 1$

$\mathcal{A}(\psi) = 1$

$\models \varphi \to \psi$

$\mathcal{A}(\varphi) = 0$

$\Downarrow$

$\max(\{\underbrace{1 - \mathcal{A}(\varphi)}_{1}, \underbrace{\mathcal{A}(\psi)}_{0}\}) = 1$

$\mathcal{A}(\psi) = 0$

($\Leftarrow$) Suppose $\models \underline{\varphi \to \psi}$ but $\varphi \not\models \psi$

There exists a Σ-valuation $\mathcal{A}$ s.t. $\mathcal{A}(\varphi) = 1$,

$\mathcal{A} \models \varphi$, $\mathcal{A}(\psi) = 0$

By definition, $\mathcal{A}(\varphi \to \psi) = \max(\{1 - \mathcal{A}(\varphi), \mathcal{A}(\psi)\})$

$= \max(\{0, 0\}) = 0$

contradiction, since we supposed $\underline{\varphi \to \psi}$

valid CNF: $(P \lor \neg P)$ $\quad$ $(\underline{P} \lor Q \lor \neg \underline{P})$

unsatisfiable DNF: $(P \land \neg P)$ $\quad$ $(\underline{P} \land \neg \underline{P} \land R)$

$$\mathcal{A}(P \lor \neg P) = \max(\{\mathcal{A}(P), \mathcal{A}(\neg P)\})$$
$$= \max(\{\mathcal{A}(P), 1 - \mathcal{A}(P)\}) = 1$$

$$\mathcal{A}(P \land \neg P) = \min(\{\mathcal{A}(P), \mathcal{A}(\neg P)\})$$
$$= \min(\{\mathcal{A}(P), 1 - \mathcal{A}(\neg P)\}) = 0$$

$$\varphi = P \leftrightarrow (Q \land \neg R)$$

$$\overset{\text{ElimEquiv}}{\underset{\text{BCNF}}{\Longrightarrow}} (P \rightarrow (Q \land \neg R)) \land ((Q \land \neg R) \rightarrow P)$$

$$\overset{\text{ElimImp}}{\underset{\text{BCNF}}{\Longrightarrow}} (\neg P \lor (Q \land \neg R)) \land (\neg(Q \land \neg R) \lor P)$$

$$\overset{\text{PushNeg2}}{\underset{\text{BCNF}}{\Longrightarrow}} (\neg P \lor (Q \land \neg R)) \land (\neg Q \lor R \lor P)$$

$$\overset{\text{PushDisj}}{\underset{\text{BCNF}}{\Longrightarrow}} (\neg P \lor Q) \land (\neg P \lor \neg R) \land (\neg Q \lor R \lor P)$$

models: $\neg P R$, $P Q \neg R$, $\neg P \neg Q \neg R$

$$P \leftrightarrow Q \quad \longleftrightarrow \quad (P \to Q) \wedge (Q \to P)$$

$$\longleftrightarrow \quad (\neg P \vee Q) \wedge (\neg Q \vee P)$$

$\mathcal{A}(P \leftrightarrow Q) = \underline{1}$ fa $\mathcal{A}(P) = \mathcal{A}(Q)$

$\psi|_P = \varphi = (P \leftrightarrow Q)$

$\mathrm{pol}(\varphi) = \underline{1}$: $\psi[R]_P \wedge (R \longrightarrow (P \to Q))$

$\mathrm{pol}(\varphi) = -1$: $\psi[R]_P \wedge ((P \to Q) \to R)$

$\mathrm{pol}(\varphi) = 0$: $\psi[R]_P \wedge (R \leftrightarrow (P \to Q))$