$$\mathcal{E} = \{\{a, b\}, \{P, R\}\}$$

$$P(x) \vee R(x, y)$$

$$\sigma_1 = \{x \to a, y \to a\}$$

$$(P(x) \vee R(x, y))\sigma_1 = P(a) \vee R(a, a)$$

$$\Sigma = \{\{a, b, g\}, \{P, R\}\}$$

$$\sigma_2 = \{x \to g(a), y \to g(g(b))\}$$

$$(P(x) \vee R(x, y))\sigma_2 = P(g(a)) \vee R(g(a), g(g(b)))$$

N sat $\Rightarrow$ N Herbrand Model

$\mathcal{A} \models N \rightsquigarrow$ Herbrand Model $\quad t_i \in T(\Sigma)$

$P^H = \{(t_1, \dots, t_n) \mid (t_1^{\mathcal{A}}, \dots, t_n^{\mathcal{A}}) \in P^{\mathcal{A}}\}$

Show $H \models N$!

by contradiction $C \in N$, grounding $C\sigma$

$H \not\models C\sigma$ for all $P(s_1, \dots, s_n) \in C\sigma$

$(s_1, \dots, s_n) \notin P^H$

$\mathcal{A} \models C\sigma$

$\rightsquigarrow L\sigma \in C\sigma$ s.t. $\mathcal{A} \models L\sigma$, L positive

by definition $L\sigma \in H$

$\Sigma = \{a, b\}$ $\quad \{ab, aba, bb\}$ $\quad I = 1, 2, 3$

$\{a, bab, abb\}$

ab aba bb

ab aba bb

$g_a, g_b \rightsquigarrow aba \rightsquigarrow R(x, y)$

$g_a(g_b(g_a(\varepsilon))) \leftarrow aba$

$R(\varepsilon, \varepsilon)$

$\neg R(x, y) \lor R(g_a(g_b(x)), g_a(y))$

$\neg R(x, y) \lor R(g_a(g_b(g_a(x))), g_b(g_a(g_b(y))))$

$\neg R(g_a(x), g_a(x)) \quad \vdots \quad \neg R(g_b(x), g_b(x))$

LIA

$$\forall x\ \exists y\ \exists z \left(\begin{array}{l} 3x + y < z \\ \vee\ x + z + y = 42 \\ \quad \ldots \end{array} \right)$$