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Tutorials for "Automated Reasoning WS20/21"
Exercise sheet 1

Exercise 1.1:
Prove by induction that for any propositional formula the number of closing parentheses is equal to the number of opening parentheses (see Definition 2.1.1).

Exercise 1.2:
Determine which of the following formulas are valid/satisfiable/unsatisfiable using propositional semantics, i.e., the definition of $\models$:

1. $\neg\big(P \lor \neg(P \land Q)\big)$
2. $(P \lor Q) \rightarrow (P \land Q)$
3. $\neg(P \rightarrow \neg P)$
4. $(P \lor \neg Q) \land \neg(\neg P \rightarrow \neg Q)$
5. $\neg(P \lor Q) \leftrightarrow (\neg P \land \neg Q)$

Exercise 1.3:
Prove the validity of the following formulas using $\Rightarrow_{\mathrm{T}}$.

1. $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$
2. $(P \rightarrow Q) \rightarrow ((R \lor P) \rightarrow (R \lor Q))$

Exercise* 1.4:
Consider a satisfiable formula ϕ with $\mathcal{A} \models \phi$.

1. Prove $\Rightarrow_{\mathrm{T}}$ to be strongly complete with respect to models: if $\{(\phi)\} \Rightarrow_{\mathrm{T}}^{*} N$ and N is a normal form then there is a sequence $(\phi, \phi_1, \ldots, \phi_n) \in N$ such that $\mathcal{A} \models \phi \land \phi_1 \land \ldots \land \phi_n$.

2. Is $\mathcal{A}$ the only model of $\phi \wedge \phi_1 \wedge \ldots \wedge \phi_n$?

Is is not encouraged to prepare joint solutions, because we do not support joint exams.