



Christoph Weidenbach

January 8, 2019

Tutorials for “Automated Reasoning WS18/19”
Exercise sheet 10

Exercise 10.1 (4.2):

Prove that the following term rewrite system is confluent:

$$\begin{aligned} f(g(x)) &\rightarrow x \\ g(f(x)) &\rightarrow x \\ f(b) &\rightarrow c \\ b &\rightarrow g(c) \end{aligned}$$

Exercise 10.2 (4.3):

Is the rewrite system

$$\{ f(a) \rightarrow f(b), f(b) \rightarrow f(c), f(c) \rightarrow f(a), f(x) \rightarrow x \}$$

(i) terminating, (ii) normalizing, (iii) locally confluent, (iv) confluent? Give a brief explanation.

Exercise 10.3 (4.6):

Let $E = \{ f(g(x)) \approx g(f(x)) \}$. Give a derivation for $E \Rightarrow_E^* f(f(g(g(y)))) \approx g(g(f(f(y))))$.

Exercise* 10.4 (4.12):

Find a signature Σ containing at least one constant symbol, a set E of Σ -equations, and two terms $s, t \in T(\Sigma, \mathcal{X})$ such that

$$T(\Sigma, \{x_1\})/E \models \forall \vec{x}(s \approx t),$$

but

$$T(\Sigma, \{x_1, x_2\})/E \not\models \forall \vec{x}(s \approx t)$$

where \vec{x} consists of all the variables occurring in s and t . The variables in \vec{x} need not be contained in $\{x_1, x_2\}$.

It is not encouraged to prepare joint solutions, because we do not support joint exams.