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Tutorials for "Automated Reasoning WS20/21"
Exercise sheet 11

Exercise 11.1:
Refute the following set of clauses via Superposition. You may freely choose an ordering and selection function and apply the well-known simplification rules. As usual variables in different clauses are different, x, y, z denote variables and f, g functions and a is a constant.

$g(\_) \leq 0$
$f(\_) \hat{=} 1$
$f > g$

1 $R(g(x), g(y), g(y), g(x))$

2 $\neg R(x, y, z, g(a)) \lor R(x, y, z, f(a))$

3 $\neg R(x, y, g(a), f(a)) \lor R(x, y, f(a), g(a))$

4 $\neg R(x, g(a), f(a), f(a)) \lor R(x, f(a), g(a), g(a))$

5 $\neg R(g(a), f(a), f(a), f(a)) \lor R(f(a), g(a), g(a), g(a))$

6 $\neg R(f(a), f(a), f(a), f(a))$

$\neg R(x, y, g(a), g(a)) \lor R(x, y, f(a), g(a))$ *
$\neg R(x, y, g(a), g(a)) \lor \neg R(x, y, f(a), f(a))$

Exercise 11.2:
Consider the below clause set N over predicate R, function g and constant a with respect to an LPO with precedence $g \succ R \succ a$. As usual one sort for everything and x, y are variables.

$a, g(a), \cancel{g(g(a))}, \ldots$ $g(a)$

1 $\neg R(x, y) \lor R(y, x)$ 2 $\neg R(x, x)$ 3 $R(x, g(x))$

4 $\neg R(g(a), a)$

$\prec g(g(a))$

a). Compute $N_{\mathcal{I}}^{\prec R(g(a),g(a))}$ and determine the minimal false clause.

b). Do the respective superposition inference with the minimal false clause, add it to N giving N' and recompute $(N')_{\mathcal{I}}^{\prec R(g(a),g(a))}$.

$\neg R(a, a) \lor R(a, a)$
$\neg R(a, a)$
$R(a, g(a))$
$\neg R(g(a), a)$
$\neg R(a, g(a)) \lor R(g(a), a)$ *
$\neg R(g(a), a) \lor R(a, g(a))$ *

Exercise 11.3:

Apply Knuth-Bendix completion ($\Rightarrow_{\text{KBC}}$) to the following set of equations with respect to a KBO where all signature symbols (and variables) have weight 1 and $f \succ g \succ b \succ a$.

$$E = \{f(g(x), y) \approx f(x, y), f(g(a), a) \approx f(b, a), g(g(x)) \approx g(x)\}$$

Exercise 11.4:
Convert the following formula in CNF using $\Rightarrow_{ACNF}$.: $P \wedge \neg[(Q \leftrightarrow R) \vee (S \rightarrow T)]$

Exercise 11.5:
Prove by congruence closure that the following ground equations are unsatisfiable: $f(a, g(a)) \approx f(b, g(b))$, $g(a) \approx h(c)$, $h(d) \approx g(b)$, $d \approx c$, $f(a, h(d)) \approx f(h(d), a)$, $f(b, g(b)) \not\approx f(h(c), a)$.

Exercise 11.6:
Check whether the following clause set is satisfiable via CDCL(LRA), where you may use the Fourier-Motzkin procedure for the linear rational arithmetic (LRA) part.

A ∨ B, C, D, E

$$N = \{\overbrace{y < 5 + x}^{A} \vee \overbrace{y > 5 + x}^{B},\ \overbrace{2x \approx z + 3}^{C},\ \overbrace{y \leq 3x + 2 - z}^{D},\ \overbrace{y - 11 + 3x \geq 2z}^{E}\}$$

[A, C, D, E]
$y < 5 + x$
$2x = z + 3$
$y \leq 3x + 2 - z$
$y - 11 + 3x \geq z$

Simp $2x \leq z + 3$, $2x \geq z + 3$ | $x = \frac{1}{2}(z + 3)$

Exercise 11.7:
Which of the following statements are true or false? Provide a proof or a counter example.

Yes

a). Let N be a first-order clause set without equality containing a clause $C \vee P(t)$. If all resolvents on $P(t)$ result in tautologies, then the clause $C \vee P(t)$ can be removed from N preserving satisfiability of N.

No
PCP

b). Let E be a set of equations. If all equations in E are orientable by an instance of the KBO, then $\Rightarrow_{\text{KBC}}$ terminates on E.

No
$f(x, a)$
$f(b, y)$

c). Let $t = f(t_1, \ldots, t_n)$ and $s = f(s_1, \ldots, s_n)$ be two terms such that the t_i, s_i are either variables or constants. Then the unification problem $E = \{s = t\}$ has a solution iff there exists a substitution σ such that $t\sigma = s$ or $s\sigma = t$.

Exercise 11.8:
A term t is called *linear* if any variable occurs at most once in t. Consider a unification problem $E = \{s = t\}$ where s, t are both linear and don't share variables. Prove the following: if E' is a normal form of E, i.e., $E \Rightarrow^*_{\text{SU}} E'$, then the size of E', i.e., the number of function, constant and variable symbols, can be linarly bound in the size of E.

Is is not encouraged to prepare joint solutions, because we do not support joint exams.

SU Substitute $E\{x \to t\} = E$
invariant: every variable at most once