Exercise 13.1:

(1) $\underline{f}(x_1, \underline{f}(x_1, x_2)) \longrightarrow g(x_2, x_1)$

(2) $g(\underline{f}(y_1, y_2), y_2) \longrightarrow g(y_1, y_2)$

Critical Pairs:

From (1) and (2) at position 1 of (2):

$mgu\left(f(y_1, y_2), f(x_1, f(x_1, x_2))\right) = \{y_1 \mapsto x_1, y_2 \mapsto f(x_1, x_2)\}$

$\langle g(x_1, f(x_1, x_2)), g(g(x_2, x_1), f(x_1, x_2)) \rangle$

From (1) and (1) at position 2 of (1):

$mgu\left(f(x_1, x_2), f(z_1, f(z_1, z_2))\right) = \{x_1 \mapsto z_1, x_2 \mapsto f(z_1, z_2)\}$

$\langle g(f(z_1, z_2), z_1), f(z_1, g(z_2, z_1)) \rangle$

Ex 13.2: $E = \{(1)\, f(x, f(x,y)) \approx g(y,x),\ (2)\, f(x,x) \approx x\}$

1. we $(z) = 1$ for $z \in \{x, y, f, g\}$ and $f > g$

$\Rightarrow f(x, f(x,y)) > g(y,x)$

$(E, \varnothing) \xRightarrow[\text{KBC}]{\text{Orient}^*} (\varnothing, \{(3)\, f(x, f(x,y)) \to g(y,x),\ (4)\, f(x,x) \to x\})$

$\left|\begin{array}{l}\text{Critical pairs:}\\ \langle g(f(x,y),x),\\ \quad f(x, g(y,x))\rangle\\ \langle g(x,x),\\ \quad f(x,x)\rangle\end{array}\right.$

$\xRightarrow[\text{KBC}]{\text{Deduce}} ((5)\, g(x,x) \approx f(x,x),\ \{(3)\},\ \{(4)\})$

$\xRightarrow[\text{KBC}]{\text{Simp.Eq }(4),(5)} ((6)\, g(x,x) \approx x,\ \{(3)\},\ \{(4)\})$

$\xRightarrow[\text{KBC}]{\text{orient}} (\varnothing,\ \{(3),\ (4),\ (7)\, g(x,x) \to x\})$

$\xRightarrow[\text{KBC}]{\text{Deduce}} (\{(8)\, g(f(x,y),x) \approx f(x, g(y,x))\},\ \{(3)\},\ (4),\ (7)\})$

$\xRightarrow[\text{KBC}]{\text{orient}} (\varnothing,\ \{(3),\ (4),\ (7),\ (9)\, g(f(x,y),x) \to f(x, g(y,x))\})$

$\left[\begin{array}{l}\text{Critical pairs:}\\ \langle g(x,x), f(x, g(x,x))\rangle\ ,\ \langle f(x, g(f(x,y),x),\ g(g(y,x),x)\rangle\end{array}\right.$

$\xRightarrow[\text{KBC}]{\text{Deduce}} ((10)\, f(x, g(f(x,y),x)) \approx g(g(y,x),x)\},\ \{\ldots\})$

$\xRightarrow[\text{KBC}]{\text{Simplify Eq }(9)(10)} ((11)\, f(x, f(x, g(y,x))) \approx g(g(y,x),x)\},\ \{\ldots\})$

$\xRightarrow[\text{KBC}]{\text{simplify-Eq }(3),(11)} ((12)\, g(g(y,x),x) \approx g(g(y,x),x)\},\ \{\ldots\})$

$\Longrightarrow_{KDL}^{Delete}$ $(\emptyset, \{(3], (4), (7), (9|\})$

$\Longrightarrow^{Deduce}$ $(\{(13)\ g(x, x) \approx f(x, g(x, x))\}, \{\ldots\})$

$\Longrightarrow^{Simplify-\bar{D}=q^{*}}$ $(4), (7), (13\ |\{(14)\ x \approx x\}, \{\ldots\})$

$\Longrightarrow_{KBC}^{Delete}$ $(\emptyset, \underbrace{\{(3), (4), (7), (9|\}}_{= R^{*}\ is\ our\ result})$

Ex 13.2.2

$g > f$ and $w(x) = 1, w(y) = 1, w(f) = 1, w(g) = 3$

$(E, \emptyset)$

$= \int \text{onduct}^{*} \underset{KBC}{} (\emptyset, \underbrace{\{ (1)\ g(y,x) \rightarrow f(x, f(x,y)),\ (2)\ f(x,x) \rightarrow x \}}_{= R^{*}})$

No critical pairs !

So R^{*} is our solution

Ex 13.3

$$N = \{ (1)\ \underline{f(x) \not\approx a} \ \lor \ f(x) \approx b,$$
$$(2)\ \underline{f(f(x)) \approx x} \ ;$$
$$(3)\ \underline{a \not\approx b} \ \}$$

LPO

$$f > a > b$$

$\underline{N}$, SupLeft (1.1, 2.1) $N \cup \{ (4)\ x \not\approx a \ \lor \ \underline{f(f(x)) \approx b} \}$

$\implies$ SupRight (2.1, 4.2) $N_1 \cup \{ (5)\ x \not\approx a \ \lor \ x \approx b \}$

$\implies$ EqRes (5.1) $N_2 \cup \{ (6)\ a \approx b \}$

$\implies$ SupLeft (3.1, 6.1) $N_3 \cup \{ (7)\ b \not\approx b \}$

$\implies$ EqRes (7.1) $N_4 \cup \{ (8)\ \bot \}$

This means N is unsat.

Ex 13.6

$N = \{$ (1) $\underline{f(a,b) \simeq b}$,

(2) $b \simeq a \lor \underline{b \simeq g(a)}$,

(3) $\underline{b \not\simeq g(b)}$,

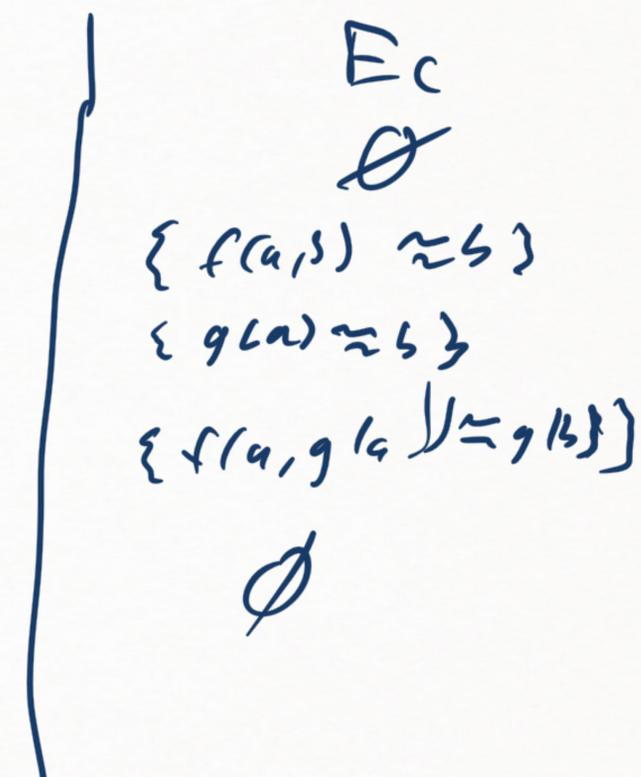(4) $\overline{f(a, g(a)) \simeq g(b)}$

(5) $\underline{b \not\simeq a}$

with an LPO $g > f > b > a$

$(5) < (1) < (2) < (4) < (3)$

2) (3) is our min. false clause

because $g(b) \downarrow^{N_I} b$

1) $\qquad\qquad C$

(5) $\qquad b \not\simeq a$

(1) $f(a,b) \simeq b$

(2) $b \simeq a \lor \underline{b \simeq g(a)}$

(4) $f(a, g(a)) \simeq g(b)$

(3) $b \not\simeq g(b)$

$\Rightarrow N_I = \{ f(a,b) \simeq b, \; g(a) \simeq b, \; f(a, g(a)) \simeq g(b) \}$

E_C

$\varnothing$

$\{ f(a,b) \simeq b \}$

$\{ g(a) \simeq b \}$

$\{ f(a, g(a)) \simeq g(b) \}$

$\varnothing$

Ex. 13.4

3) $N \Rightarrow$ Sup Left((3.1, 4.1) $N \cup \{ (6)\ b \not\approx f(a, g(a)) \} = N'$

(5) < (1) < (2) < (6) < (4) < (3)

C		EC
(5)	$b \not\approx a$	$\varnothing$
(1)	$f(a, b) \approx b$	$\{ f(a, b) \approx b \}$
(2)	$b \not\approx a \lor \underline{b \approx g(a)}$	$\{ b \approx g(a) \}$
(6)	$b \not\approx f(a, (g(a))$	$\varnothing$
(4)	$f(a, g(a)) \approx g(b)$	$\{ g(b) \approx f(a, g(a)) \}$
(3)	$b \not\approx g(b)$	$\varnothing$

$N'_I \approx N_I$

und (6) is the mini false clause