max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

Bromberger/Möhle/Schwarz/Weidenbach February 2, 2023

Tutorials for "Automated Reasoning WS22/23"
Exercise sheet 14

This exercise sheet is prototypical for the final exam!

Exercise 14.1:
Use CDCL to decide satisfiability of the following clause set.

(1)	$P_1 \lor P_2 \lor P_3$	(2)	$P_4 \lor P_5 \lor P_6$	(3)	$\neg P_3 \lor P_5$
(4)	$\neg P_1 \lor \neg P_4$	(5)	$\neg P_2 \lor \neg P_5$	(6)	$\neg P_3 \lor \neg P_6$
(7)	$\neg P_6$	(8)	$\neg P_5 \lor P_2$		

Exercise 14.2:
Consider the following clause set N with respect to an LPO where $g \succ f \succ b \succ a$.

$$N = \{f(a, b) \approx b, b \approx a \lor b \approx g(a), b \not\approx g(b), f(a, g(a)) \approx g(b), b \not\approx a\}$$

1. Compute $N_{\mathcal{I}}$.
2. Determine the minimal false clause.
3. Compute the superposition inference out of 2., add it to the clause set N compute the new respective $N_{\mathcal{I}}$.

Exercise 14.3:
Refute the following clause set by superposition. Choose an appropriate ordering and selection strategy.

$$N = \{f(x, y) \approx g(y, x) \lor R(x, y),\ \neg R(g(x, y), a),\ R(f(g(x, y), y), y),\ \neg R(g(a, x), b) \lor f(x, a) \not\approx g(x, a)\}$$

Exercise 14.4:
Use the congruence closure algorithm to check whether the following conjunction of equations

$$f(f(a)) \approx a \wedge f(a) \approx b \wedge f(f(b)) \approx g(b) \wedge a \not\approx b \wedge h(a,b) \not\approx h(a,g(b))$$

is false.

Exercise 14.5:
Consider the below set of inequations and apply the simplex algorithm to it:

$$\begin{array}{rcl} 2x+5y & \leq & -17 \\ 3x+7y & \leq & -24 \\ 2x+5y & \geq & -17 \\ 3x+7y & \geq & -24 \end{array}$$

Exercise 14.6:
Prove or refute by counterexample the following statements:

1. If ϕ is a first-order formula and x a variable, then ϕ is unsatisfiable if and only if $\exists x.\phi$ is unsatisfiable.
2. If ϕ and ψ are first-order formulas and x is a variable, then $\forall x.(\phi \wedge \psi) \models (\forall x.\phi) \wedge (\forall x.\psi)$ and $(\forall x.\phi) \wedge (\forall x.\psi) \models \forall x.(\phi \wedge \psi)$.
3. If ϕ and ψ are first-order formulas and x is a variable, then $\exists x.(\phi \wedge \psi) \models (\exists x.\phi) \wedge (\exists x.\psi)$ and $(\exists x.\phi) \wedge (\exists x.\psi) \models \exists x.(\phi \wedge \psi)$.

Exercise 14.7:
Let $\phi = P(f(c)) \wedge \forall x.(P(x) \rightarrow P(f(x)))$ and $\psi = \forall x.P(f(f(x)))$ be first-order formulas. Prove that every Herbrand model of ϕ considering the signature of ϕ only is also a model of ψ.

It is not encouraged to prepare joint solutions, because we do not support joint exams.