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Tutorials for "Automated Reasoning WS20/21"
Exercise sheet 2

Exercise 2.1:
Convert the following formulas in CNF using both $\Rightarrow_{BCNF}$ and $\Rightarrow_{ACNF}$.:

1. $[(P \rightarrow S) \wedge \neg Q] \leftrightarrow [R \vee (\neg S \rightarrow Q)]$
2. $[\neg(\neg P \vee (Q \wedge R))] \rightarrow [P \wedge (\neg Q \leftrightarrow \neg R)]$
3. $\neg[(P \wedge (P \rightarrow Q)) \leftrightarrow (P \vee Q)]$

Exercise 2.2:
Prove that the following formula is valid via resolution:

$$(P \rightarrow Q) \rightarrow [(R \vee P) \rightarrow (R \vee Q)]$$

apply $\Rightarrow_{ACNF}$ to the negated formula and the resolution calculus to the resulting clauses until you derive the empty clause.

Exercise 2.3:
We call a set N of clauses exhausted if the result of any inference with clauses from N is already in the set N or is subsumed by a clause in N. Compute an exhausted equivalent set of clauses for: $\{\neg P \vee Q \vee \neg S, \neg P \vee Q \vee S, P \vee S, P \vee \neg Q \vee \neg S, \neg P \vee \neg Q \vee \neg S, Q \vee \neg S \vee P\}$ by using $\Rightarrow_{\text{RES}}$.

Exercise* 2.4:
Let N be a finite set of propositional clauses and P a propositional variable. Assume that we don't have duplicate literals in clauses and that no clause contains Q and $\neg Q$ for any propositional variable Q. Let $P \vee C_1, \ldots, P \vee C_k$ be all clauses in N containing the literal P and $\neg P \vee D_1, \ldots, \neg P \vee D_l$ be all clauses in N containing literal $\neg P$. Define the set $\mathcal{E}(P, N) = (N - \{P \vee C_i \mid 1 \leq i \leq k\} - \{\neg P \vee D_j \mid 1 \leq j \leq l\}) \cup \{C_i \vee D_j \mid 1 \leq i \leq k, 1 \leq j \leq l\}$. Prove that N is satisfiable iff $\mathcal{E}(P, N)$ is satisfiable.

Is is not encouraged to prepare joint solutions, because we do not support joint exams.