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**Tutorials for "Automated Reasoning WS20/21"**
**Exercise sheet 3**

**Exercise 3.1:**
Use CDCL to decide satisfiability of the following clause set.

$$
\begin{array}{llll}
(1) & \neg P_1 \vee \neg P_2 & (2) & P_3 \vee P_2 \vee P_4 & (3) & P_2 \vee \neg P_4 \\
(4) & \neg P_3 \vee P_2 & (5) & P_1 \vee P_2 \vee P_4
\end{array}
$$

**Exercise 3.2:**
Use CDCL to decide satisfiability of the following clause set.

$$
\begin{array}{llll}
(1) & P_1 \vee P_2 \vee P_3 & (2) & P_4 \vee P_5 \vee P_6 & (3) & \neg P_3 \vee P_5 \\
(4) & \neg P_1 \vee \neg P_4 & (5) & \neg P_2 \vee \neg P_5 & (6) & \neg P_3 \vee \neg P_6 \\
(7) & \neg P_6 & (8) & \neg P_5 \vee P_2
\end{array}
$$

**Exercise 3.3:**
Demonstrate the partial model construction of propositional superposition on the following sets of clauses:

1. Set of clauses $N = \{\neg Q_0 \vee \neg P_2 \vee Q_1, \neg Q_1 \vee Q_2, P_0 \vee Q_0, \neg Q_0 \vee P_1, Q_0 \vee P_1\}$. Use ordering $Q_2 \succ P_2 \succ Q_1 \succ P_1 \succ Q_0 \succ P_0$ on atoms.

2. Set of clauses $N = \{\neg P \vee Q \vee P, S \vee \neg Q \vee R, \neg R \vee \neg S, Q \vee \neg S \vee S, R \vee S \vee P, S \vee Q, \neg R \vee \neg P \vee S \vee \neg Q\}$. Use ordering $P \succ Q \succ R \succ S$ on atoms.

Demonstrate here means: order the clauses in the set, show how (partial) interpretations (i.e. $N_D$ for every $D \in N$) looks like, show how $\delta_D$ look like for every $D \in N$ and show the minimal clause which is not entailed by $N_\mathcal{I}$ if there is some. Don't do any inferences!

**Exercise 3.4:**
Show unsatisfiability of the below clause set $N$ via the propositional superposition calculus

based on the atom ordering $P_1 \succ P_4 \succ P_5 \succ P_2 \succ P_3$.

$$
\begin{array}{llllll}
(1) & P_1 \vee P_2 \vee P_3 & (2) & \neg P_1 \vee \neg P_2 & (3) & \neg P_2 \vee \neg P_3 \\
(4) & \neg P_1 \vee \neg P_3 & (5) & P_4 \vee P_5 \vee P_1 & (6) & \neg P_4 \vee P_1 \\
(7) & \neg P_4 \vee P_2 & (8) & \neg P_5 \vee P_2 & (9) & \neg P_5 \vee P_3 \\
(10) & \neg P_1 \vee P_4 & & & &
\end{array}
$$

**Exercise\* 3.5:**
Consider the following clause set

$$
\begin{array}{llllll}
(1) & P_1 \vee P_2 \vee P_3 & (2) & P_1 \vee \neg P_2 & (3) & \neg P_1 \vee \neg P_2 \\
(4) & \neg P_1 \vee P_2 & (5) & P_4 \vee P_5 \vee \neg P_3 & (6) & P_4 \vee \neg P_5 \\
(7) & \neg P_4 \vee \neg P_5 & (8) & \neg P_4 \vee P_5 & &
\end{array}
$$

and the resolution refutation of the clause set

$$
\begin{array}{lllll}
(2)R(3) & \Rightarrow_{\mathrm{RES}} (9) & \neg P_2 & (9)R(1) & \Rightarrow_{\mathrm{RES}} (10) \quad P_1 \vee P_3 \\
(3)R(4) & \Rightarrow_{\mathrm{RES}} (11) & \neg P_1 & (11)R(10) & \Rightarrow_{\mathrm{RES}} (12) \quad P_3 \\
(12)R(5) & \Rightarrow_{\mathrm{RES}} (13) & P_4 \vee P_5 & (6)R(7) & \Rightarrow_{\mathrm{RES}} (14) \quad \neg P_5 \\
(14)R(13) & \Rightarrow_{\mathrm{RES}} (15) & P_4 & (7)R(8) & \Rightarrow_{\mathrm{RES}} (16) \quad \neg P_4 \\
(15)R(16) & \Rightarrow_{\mathrm{RES}} (17) & \bot & &
\end{array}
$$

Present a CDCL run that first learns clause (9) and then clause (10). You may use rule Restart to this end.

Is is not encouraged to prepare joint solutions, because we do not support joint exams.