



Christoph Weidenbach

December 2, 2020

Tutorials for “Automated Reasoning WS20/21”
Exercise sheet 4

Exercise 4.1:

Let $a : \rightarrow S$ and $R \subseteq S \times T$. Complete the sort information for g, f, P and variables x, y such that the following formula is well-sorted: $\forall x, y. (R(x, g(x)) \rightarrow (f(g(x), a) \approx y \vee P(y) \vee R(x, y)))$

Exercise 4.2:

Check whether the following first-order formulas are satisfiable, valid or unsatisfiable, where a and b are constants and g is a unary function symbol. Assume a one-sorted universe.

1. $(\forall x. \exists y. R(x, y)) \rightarrow R(a, b)$
2. $(P(a) \wedge \forall x. (P(x) \rightarrow P(g(x)))) \rightarrow P(g(g(a)))$
3. $(\exists x. P(x)) \rightarrow P(b)$
4. $P(b) \rightarrow (\exists x. P(x))$

Exercise 4.3:

Use the FM method to decide whether the following conjunction of inequations is satisfiable:

- $$\begin{aligned} x + y &\geq 16 & (1) \\ 4x + 7y &\leq 28 & (2) \\ 2x - 7y &\leq 20 & (3) \\ 2x - 3y &\geq -9 & (4) \end{aligned}$$

Exercise 4.4:

Check via FM whether the following formulas are true/false:

1. $\forall x.\exists y.(2x + y > 7 \wedge x + y < 6)$
2. $\exists x.\forall y.(2x - y > 7 \wedge 2x + y > 7)$

Exercise* 4.5:

Provide first-order formulas such that the domain of any interpretation satisfying the formula

1. has exactly 3 elements
2. is infinite

It is not encouraged to prepare joint solutions, because we do not support joint exams.