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Tutorials for "Decision Procedures SS19"
Exercise sheet 8

Exercise 8.1:
Let $\Lambda_1 \parallel C_1 \vee L_1$ and $\Lambda_2 \parallel C_2 \vee L_2$ be two BS clauses with simple bounds. Let σ be the unifier of L_1 and $\text{comp}(L_2)$. Prove that $(\Lambda_1, \Lambda_2 \parallel C_1 \vee C_2)\sigma$ is a BS clause with simple bounds. This might need some simplification.

Exercise 8.2:
For the pure BS fragment the following rules are sound, complete, and terminate:

Superposition-BS $M \uplus \{N \uplus \{P(t_1, \ldots, t_n), C \vee \neg P(s_1, \ldots, s_n)\}\} \Rightarrow_{\text{SUPBS}} M \cup \{N \cup \{P(t_1, \ldots, t_n), C \vee \neg P(s_1, \ldots, s_n)\} \cup \{C\sigma\}\}$

where (i) $\neg P(s_1, \ldots, s_n)$ is selected in $(C \vee \neg P(s_1, \ldots, s_n))\sigma$ (ii) σ is the mgu of $P(t_1, \ldots, t_n)$ and $P(s_1, \ldots, s_n)$ (iii) $C \vee \neg P(s_1, \ldots, s_n)$ is a Horn clause

Instantiation $M \uplus \{N \uplus \{C \vee A_1 \vee A_2\}\} \Rightarrow_{\text{SUPBS}} M \cup \{N \cup \{(C \vee A_1 \vee A_2)\sigma_i \mid \sigma_i = \{x \mapsto a_i\}, 1 \leq i \leq k\}\}$

where x occurs in a variable chain between A_1 and A_2

Split $M \uplus \{N \uplus \{C_1 \vee A_1 \vee C_2 \vee A_2\}\} \Rightarrow_{\text{SUPBS}} M \cup \{N \cup \{C_1 \vee A_1\}, N \cup \{C_2 \vee A_2\}\}$

where $\text{vars}(C_1 \vee A_1) \cap \text{vars}(C_2 \vee A_2) = \emptyset$

Discuss to what extend the rules can be turned into a sound, complete and terminating calculus for BS with simple bounds over LRA.