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Tutorials for "Automated Reasoning WS20/21"
Exercise sheet 9

Exercise 9.1:
Consider the below clause set N, $\Sigma = (\{S\}, \{g, b, a\}, \{P, R\})$, with a KBO ordering where all signature symbols and variables have weight 1 and atoms are compared like terms with precedence $P \succ R \succ g \succ b \succ a$.

$$\begin{array}{ll} 1 & \neg P(x) \vee P(g(x)) \\ 2 & \neg P(x) \vee R(x, g(x)) \\ 3 & P(a) \vee P(b) \\ 4 & \neg R(b, g(b)) \vee P(a) \end{array}$$

1. Compute $\mathrm{grd}(\Sigma, N)^{\prec \neg R(b,g(b)) \vee P(b)}$, i.e., generate all ground instances of N smaller than $\neg R(b, g(b)) \vee P(b)$ and run the partial model operator.
2. Determine the minimal false ground clause and its productive counterpart and perform the superposition inference step on the respective first-order clauses from N, not on the ground instances, resulting in the clause set N'.
3. Run the partial model operator on $\mathrm{grd}(\Sigma, N')^{\prec \neg R(b,g(b)) \vee P(b)}$. Can the resulting partial model be extended to a model for N' by adding further (arbitrarily chosen) ground atoms? If no, provide an argument why there is always at least one false clause for any extension, if yes provide the complete model and give an argument why it is a model.
4. Consider the above three steps once more after adding the clause $\neg P(g(b))$ to N.

Exercise 9.2:
Define a selection function sel and ordering $\succ$ such that the following set of clauses N is saturated with respect to the calculus $\Rightarrow_{\mathrm{SUP}}$. $N = \{\neg P(x) \vee Q(f(a)) \vee P(f(x)),\ P(g(x)) \vee P(g(f(x))) \vee \neg P(a),\ P(g(x)) \vee Q(y) \vee P(f(f(f(y)))) \vee \neg Q(f(y)),\ P(g(y)) \vee Q(f(a)),\ Q(f(z)) \vee P(f(g(f(z)))) \vee \neg Q(f(g(f(z))))\}$. Justify your definition of the selection function.

Exercise 9.3:
Compute all critical pairs for each of the following systems:

1. $R = \{f(g(f(x))) \to x,\ f(g(x)) \to g(f(x))\}$;
2. $R = \{0 + y \to y,\ s(x) + y \to s(x + y),\ x + 0 \to x,\ x + s(y) \to s(x + y)\}$;

Note: recall that variables in rewrite systems are universally quantified, hence they can be renamed such that no two rules of the same system share a variable.

Exercise* 9.4:
Is the following statements are true or false? Provide a proof or a counter example.

1. If N is a non-empty and saturated set of first-order clauses then each clause in N has a unique maximal (or selected) literal.

Is is not encouraged to prepare joint solutions, because we do not support joint exams.