

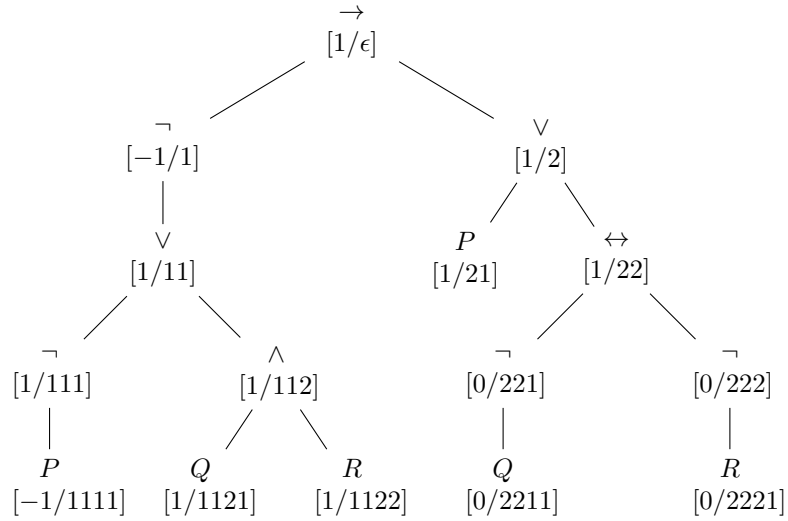
Figure 2.6: Semantic tableau.

ElimEquiv	$\chi[(\phi \leftrightarrow \psi)]_p \Rightarrow_{\text{BCNF}} \chi[(\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)]_p$
ElimImp	$\chi[(\phi \rightarrow \psi)]_p \Rightarrow_{\text{BCNF}} \chi[(\neg\phi \vee \psi)]_p$
PushNeg1	$\chi[\neg(\phi \vee \psi)]_p \Rightarrow_{\text{BCNF}} \chi[(\neg\phi \wedge \neg\psi)]_p$
PushNeg2	$\chi[\neg(\phi \wedge \psi)]_p \Rightarrow_{\text{BCNF}} \chi[(\neg\phi \vee \neg\psi)]_p$
PushNeg3	$\chi[\neg\neg\phi]_p \Rightarrow_{\text{BCNF}} \chi[\phi]_p$
PushDisj	$\chi[(\phi_1 \wedge \phi_2) \vee \psi]_p \Rightarrow_{\text{BCNF}} \chi[(\phi_1 \vee \psi) \wedge (\phi_2 \vee \psi)]_p$
PushConj	$\chi[(\phi_1 \vee \phi_2) \wedge \psi]_p \Rightarrow_{\text{BDNF}} \chi[(\phi_1 \wedge \psi) \vee (\phi_2 \wedge \psi)]_p$
ElimTB1	$\chi[(\phi \wedge \top)]_p \Rightarrow_{\text{BCNF}} \chi[\phi]_p$
ElimTB2	$\chi[(\phi \wedge \perp)]_p \Rightarrow_{\text{BCNF}} \chi[\perp]_p$
ElimTB3	$\chi[(\phi \vee \top)]_p \Rightarrow_{\text{BCNF}} \chi[\top]_p$
ElimTB4	$\chi[(\phi \vee \perp)]_p \Rightarrow_{\text{BCNF}} \chi[\phi]_p$
ElimTB5	$\chi[\neg\perp]_p \Rightarrow_{\text{BCNF}} \chi[\top]_p$
ElimTB6	$\chi[\neg\top]_p \Rightarrow_{\text{BCNF}} \chi[\perp]_p$

Figure 2.7: Basic CNF/DNF Transformation Rules

$$\begin{aligned}
& \neg((P \vee Q) \leftrightarrow (P \rightarrow (Q \wedge \top))) \\
\Rightarrow_{\text{Step 1}} & \neg([(P \vee Q) \rightarrow (P \rightarrow (Q \wedge \top))] \wedge [(P \rightarrow (Q \wedge \top)) \rightarrow (P \vee Q)]) \\
\Rightarrow_{\text{Step 2}} & \neg([\neg(P \vee Q) \vee (P \rightarrow (Q \wedge \top))] \wedge [(P \rightarrow (Q \wedge \top)) \rightarrow (P \vee Q)]) \\
\Rightarrow_{\text{Step 2}} & \neg([\neg(P \vee Q) \vee (P \rightarrow (Q \wedge \top))] \wedge [\neg(P \rightarrow (Q \wedge \top)) \vee (P \vee Q)]) \\
\Rightarrow_{\text{Step 2}} & \neg([\neg(P \vee Q) \vee (P \rightarrow (Q \wedge \top))] \wedge [\neg(\neg P \vee (Q \wedge \top)) \vee (P \vee Q)]) \\
\Rightarrow_{\text{Step 2}} & \neg([\neg(P \vee Q) \vee (\neg P \vee (Q \wedge \top))] \wedge [\neg(\neg P \vee (Q \wedge \top)) \vee (P \vee Q)]) \\
\Rightarrow_{\text{Step 3}} & \neg([\neg(P \vee Q) \vee (\neg P \vee Q)] \wedge [\neg(\neg P \vee Q) \vee (P \vee Q)]) \\
\Rightarrow_{\text{Step 4}} & \neg([\neg(P \wedge \neg Q) \vee (\neg P \vee Q)] \wedge [\neg(\neg P \vee Q) \vee (P \vee Q)]) \\
\Rightarrow_{\text{Step 4}} & \neg([\neg(P \wedge \neg Q) \vee (\neg P \vee Q)] \wedge [(\neg \neg P \wedge \neg Q) \vee (P \vee Q)]) \\
\Rightarrow_{\text{Step 4}} & \neg([\neg(P \wedge \neg Q) \vee (\neg P \vee Q)] \wedge [(\neg \neg P \wedge \neg Q) \vee (P \vee Q)]) \\
\Rightarrow_{\text{Step 4}} & \neg([\neg(P \wedge \neg Q) \vee (\neg P \vee Q)] \wedge [(\neg \neg P \wedge \neg Q) \vee (P \vee Q)]) \\
\Rightarrow_{\text{Step 4}} & [(\neg \neg P \vee \neg \neg Q) \wedge (\neg \neg P \wedge \neg Q)] \vee [(\neg \neg \neg P \vee \neg \neg Q) \wedge (\neg P \wedge \neg Q)] \\
\Rightarrow_{\text{Step 4}} & [(P \vee Q) \wedge (P \wedge \neg Q)] \vee [(\neg P \vee Q) \wedge (\neg P \wedge \neg Q)] \\
\Rightarrow_{\text{Step 5}} & (P \vee Q \vee \neg P \vee Q) \wedge (P \vee Q \vee \neg P) \wedge (P \vee Q \vee \neg Q) \wedge (P \vee \neg P \vee Q) \wedge (P \vee \neg P) \wedge (P \vee \neg Q) \wedge (\neg Q \vee \neg P \vee Q) \wedge (\neg Q \vee \neg P) \wedge (\neg Q \vee \neg Q)
\end{aligned}$$

Figure 2.8: Example Basic CNF Transformation

Figure 2.9: Tree representation of $[\neg(\neg P \vee (Q \wedge R))] \rightarrow [P \vee (\neg Q \leftrightarrow \neg R)]$ where each node is annotated with its [polarity/position].