# The Array Fragment with Extensionality

The array first-order array theory consists of (at least) three sorts. An index sort $I$, an array sort $A$ and a value sort $V$. The function $\mathsf{store} : A \times I \times V \to A$ stores values into the array and the function $\mathsf{read} : A \times I \to V$ reads values from the array.
The array theory $\mathcal{T}_{\mathsf{Array}}$ with extensionality consists then of the three first-order axioms

$$\forall x_A, y_I, z_V.\, \mathsf{read}(\mathsf{store}(x, y, z), y) \approx z$$
$$\forall x_A, y_I, {y'}_I, z_V.(y \not\approx y' \to \mathsf{read}(\mathsf{store}(x, y, z), y') \approx \mathsf{read}(x, y'))$$
$$\forall x_A, {x'}_A.\exists y_I.(\mathsf{read}(x, y) \not\approx \mathsf{read}(x', y) \vee x \approx x')$$

Similar to the Flattening rule of congruence closure, we also assume here that by the introduction of new connstants of the appropriate sorts, there are no nested occurrences of read and store.

**FlattS** $N[\mathsf{store}(a, s, t)]_{p_1,\ldots,p_k} \Rightarrow_{\mathsf{AF}}$
$N[b/p_1, \ldots, p_k] \cup \{b \approx \mathsf{store}(a, s, t)\}$
where $b$ is fresh

**FlattR** $N[\mathsf{read}(a, s)]_{p_1,\ldots,p_k} \Rightarrow_{\mathsf{AF}} N[b/p_1, \ldots, p_k] \cup \{b \approx \mathsf{read}(a, s)\}$
where $b$ is fresh

The Flattening rules are applied at most once to each term $\mathsf{store}(a, s, t)$ or $\mathsf{read}(a, s)$, respectively.

Then the following inference rules are build in combination with a decision procedure for ground clauses over EUF, e.g., by CDCL(EUF):

**StoreRead** $N \cup \{a \approx \mathsf{store}(b, i, l)\} \Rightarrow_{\mathsf{AR}} N \cup \{\mathsf{read}(a, i) \approx l\}$

**StoreTransD** $N \cup \{a \approx \mathsf{store}(b, i, l), l' \approx \mathsf{read}(a', j)\} \Rightarrow_{\mathsf{AR}}$
$N \cup \{i \approx j \vee \mathsf{read}(a, j) \approx \mathsf{read}(b, j)\}$
if $N \models_{\mathcal{T}_{\mathsf{EUF}}} a \approx a'$

**StoreTransU** $N \cup \{a \approx \mathsf{store}(b, i, l), l' \approx \mathsf{read}(b', j)\} \Rightarrow_{\mathsf{AR}}$
$N \cup \{i \approx j \vee \mathsf{read}(a, j) \approx \mathsf{read}(b, j)\}$
if $N \models_{\mathcal{T}_{\mathsf{EUF}}} b \approx b'$

**Ext** $N \Rightarrow_{\mathsf{AR}} N \cup \{a \approx b \vee \mathsf{read}(a, i) \not\approx \mathsf{read}(b, i)\}$
if $a, b$ are arrays in $N$ and $i$ is fresh