Unification

3.7.1 Definition (Unifier)

Two terms s and t of the same sort are said to be *unifiable* if there exists a well-sorted substitution σ so that $s\sigma = t\sigma$, the substitution σ is then called a well-sorted *unifier* of s and t.

The unifier σ is called *most general unifier*, written $\sigma = mgu(s, t)$, if any other well-sorted unifier τ of s and t it can be represented as $\tau = \sigma\tau'$, for some well-sorted substitution τ'.

A state of the naive standard unification calculus is a set of equations E or $\bot$, where $\bot$ denotes that no unifier exists. The set E is also called a *unification problem*.

The start state for checking whether two terms s, t, $\text{sort}(s) = \text{sort}(t)$, (or two non-equational atoms A, B) are unifiable is the set $E = \{s = t\}$ ($E = \{A = B\}$). A variable x is *solved* in E if $E = \{x = t\} \uplus E'$, $x \not\in \text{vars}(t)$ and $x \not\in \text{vars}(E)$.

A variable $x \in \text{vars}(E)$ is called *solved* in E if $E = E' \uplus \{x = t\}$ and $x \not\in \text{vars}(t)$ and $x \not\in \text{vars}(E')$.

Standard (naive) Unification

Tautology $E \uplus \{t = t\} \Rightarrow_{\mathrm{SU}} E$

Decomposition $E \uplus \{f(s_1, \ldots, s_n) = f(t_1, \ldots, t_n)\} \Rightarrow_{\mathrm{SU}}$
$E \cup \{s_1 = t_1, \ldots, s_n = t_n\}$

Clash $E \uplus \{f(s_1, \ldots, s_n) = g(s_1, \ldots, s_m)\} \Rightarrow_{\mathrm{SU}} \bot$
if $f \neq g$

Substitution $E \uplus \{x = t\} \Rightarrow_{SU} E\{x \mapsto t\} \cup \{x = t\}$
if $x \in vars(E)$ and $x \notin vars(t)$

Occurs Check $E \uplus \{x = t\} \Rightarrow_{SU} \bot$
if $x \neq t$ and $x \in vars(t)$

Orient $E \uplus \{t = x\} \Rightarrow_{SU} E \cup \{x = t\}$
if $t \notin \mathcal{X}$