Decision Procedures for BS

3.15.3 Definition (Bernays-Schoenfikel Fragment (BS))

A formula of the Bernays-Schoenfikel fragment has the form $\exists \vec{x}.\forall \vec{y}.\phi$ such that ϕ does not contain quantifiers nor non-constant function symbols.

3.15.4 Theorem (BS is decidable)

Unsatisfiability of a BS clause set is decidable.

$$1 : \neg R(x, y) \lor \neg R(y, z) \lor R(x, z)$$
$$2 : R(x, y) \lor R(y, x)$$

A state is now a set of clause sets. Let k be the number of different constants $a_1, \ldots, a_k$ in the initial clause set N. Then the initial state is the set $M = \{N\}$, Superposition Left is adopted to the new setting, Factoring is no longer needed and the rules Instantiate and Split are added. The variables $x_1, \ldots, x_k$ constitute a *variable chain* between literals L_1, L_k inside a clause C, if there are literals $\{L_1, \ldots, L_k\} \subseteq C$ such that $x_i \in (\mathsf{vars}(L_i) \cap \mathsf{vars}(L_{i+1}))$, $1 \leq i < k$.

Superposition BS

$M \uplus \{N \uplus \{P(t_1, \ldots, t_n), C \vee \neg P(s_1, \ldots, s_n)\}\} \Rightarrow_{\text{SUPBS}}$
$M \cup \{N \cup \{P(t_1, \ldots, t_n), C \vee \neg P(s_1, \ldots, s_n)\} \cup \{C\sigma\}\}$

where (i) $\neg P(s_1, \ldots, s_n)$ is selected in $(C \vee \neg P(s_1, \ldots, s_n))\sigma$ (ii) σ
is the mgu of $P(t_1, \ldots, t_n)$ and $P(s_1, \ldots, s_n)$
(iii) $C \vee \neg P(s_1, \ldots, s_n)$ is a Horn clause

Instantiation $\qquad\qquad M \uplus \{N \uplus \{C \vee A_1 \vee A_2\}\} \Rightarrow_{\text{SUPBS}}$
$M \cup \{N \cup \{(C \vee A_1 \vee A_2)\sigma_i \mid \sigma_i = \{x \mapsto a_i\}, 1 \leq i \leq k\}\}\}$

where x occurs in a variable chain between A_1 and A_2

Split $\qquad\qquad\qquad\qquad M \uplus \{N \uplus \{C_1 \vee A_1 \vee C_2 \vee A_2\}\}$
$\Rightarrow_{\text{SUPBS}} \ M \cup \{N \cup \{C_1 \vee A_1\}, N \cup \{C_2 \vee A_2\}\}$
where $\text{vars}(C_1 \vee A_1) \cap \text{vars}(C_2 \vee A_2) = \emptyset$

3.16.1 Definition (Rigorous Selection Strategy)

A selection strategy is *rigorous* of in any clause containing a negative literal, a negative literal is selected.

3.16.2 Lemma (SUPBS Basic Properties)

The SUPBS rules have the following properties:

1. Superposition BS is sound.
2. Instantiation is sound and complete.
3. Split is sound and complete.