

A CDCL(T) problem state is a five-tuple $(M; N; U; k; C)$ where N is the propositional abstraction of some clause set N' , $N = \text{atr}(N')$, M a sequence of annotated propositional literals, U is a set of derived propositional clauses, $k \in \mathbb{N} \cup \{-1\}$, and C is a propositional clause or \top or \perp . In particular, the following states can be distinguished:

- $(\epsilon; N; \emptyset; 0; \top)$ is the start state for some clause set N
- $(M; N; U; -1; \top)$ is a final state, where $\text{atr}^{-1}(M) \models_{\mathcal{T}} N'$
- $(M; N; U; k; \perp)$ is a final state, where N' has no model
- $(M; N; U; k; \top)$ is a model search state if $k \neq 0$
- $(M; N; U; k; D)$ is a backtracking state if $D \notin \{\top, \perp\}$



Propagate $(M; N; U; k; \top) \Rightarrow_{\text{CDCL}} (ML^{C \vee L}; N; U; k; \top)$
 provided $C \vee L \in (N \cup U)$, $M \models \neg C$, and L is undefined in M

Decide $(M; N; U; k; \top) \Rightarrow_{\text{CDCL}} (ML^{k+1}; N; U; k + 1; \top)$
 provided L is undefined in M

Conflict $(M; N; U; k; \top) \Rightarrow_{\text{CDCL}} (M; N; U; k; D)$
 provided $D \in (N \cup U)$ and $M \models \neg D$

Skip $(ML^{C\vee L}; N; U; k; D) \Rightarrow_{\text{CDCL}} (M; N; U; k; D)$
 provided $D \notin \{\top, \perp\}$ and $\text{comp}(L)$ does not occur in D

Resolve $(ML^{C\vee L}; N; U; k; D \vee \text{comp}(L)) \Rightarrow_{\text{CDCL}}$
 $(M; N; U; k; D \vee C)$
 provided D is of level k

Backtrack $(M_1 K^{i+1} M_2; N; U; k; D \vee L) \Rightarrow_{\text{CDCL}}$
 $(M_1 L^{D\vee L}; N; U \cup \{D \vee L\}; i; \top)$
 provided L is of level k and D is of level i .

Restart $(M; N; U; k; \top) \Rightarrow_{\text{CDCL}} (\epsilon; N; U; 0; \top)$
 provided $M \not\models N$

Forget $(M; N; U \uplus \{C\}; k; \top) \Rightarrow_{\text{CDCL}} (M; N; U; k; \top)$
 provided $M \not\models N$

Note that these rules are exactly the rules of CDCL from Section 2.9. The only difference that any normal form $(M; N; U; k; \top)$ was a final state in CDCL, but not in CDCL(\mathcal{T}) because $k \neq -1$. On the other hand, if CDCL derives the empty clause, i.e., \perp , then this is also a final state for CDCL(\mathcal{T}), see Lemma 7.2.1. The \mathcal{T} rules are missing that in particular check whether the propositional model is in fact also a theory model.



\mathcal{T} -Success $(M; N; U; k; \top) \Rightarrow_{\text{CDCL}(\mathcal{T})} (M; N; U; -1; \top)$
 provided $M \models (N \cup U)$ and $\text{atr}^{-1}(M)$ is \mathcal{T} -satisfiable

\mathcal{T} -Propagate $(M; N; U; k; \top) \Rightarrow_{\text{CDCL}(\mathcal{T})} (ML^{C \vee L}; N; U; k; \top)$
 provided $\text{atr}^{-1}(M)$ is \mathcal{T} -satisfiable, L is undefined in M but
 $\text{atom}(L)$ occurs in $N \cup U$, and there are literals L_1, \dots, L_n from M
 with $\text{atr}^{-1}(L_1), \dots, \text{atr}^{-1}(L_n) \models_{\mathcal{T}} \text{atr}^{-1}(L)$ and
 $C = \text{comp}(L_1) \vee \dots \vee \text{comp}(L_n)$

\mathcal{T} -Conflict $(M; N; U; k; \top) \Rightarrow_{\text{CDCL}(\mathcal{T})}$
 $(\epsilon; N; U \cup \{\neg L_1 \vee \dots \vee \neg L_n\}; 0; \top)$
 provided there are literals L_1, \dots, L_n from M with
 $\text{atr}^{-1}(L_1), \dots, \text{atr}^{-1}(L_n) \models_{\mathcal{T}} \perp$