2.9.11 Proposition (CDCL Strong Completeness)

The CDCL rule set is complete: for any valuation M with $M \models N$ there is a reasonable sequence of rule applications generating $(M'; N; U; k; \top)$ as a final state, where M and M' only differ in the order of literals.

2.9.12 Proposition (CDCL Termination)

Assume the algorithm CDCL with all rules except Restart and Forget is applied using a reasonable strategy. Then it terminates in a state $(M; N; U; k; D)$ with $D \in \{\top, \bot\}$.

The Overall Picture

Application System + Problem
System Algorithm + Implementation
Algorithm Calculus + Strategy
Calculus Logic + States + Rules
Logic Syntax + Semantics

Algorithm: 5 CDCL(S)

Input : An initial state $(\epsilon; N; \emptyset; 0; \top)$.

Output: A final state $S = (M; N; U; k; \top)$ or $S = (M; N; U; k; \bot)$

```
while (any rule applicable) do
    ifrule (Conflict(S)) then
        while (Skip(S) || Resolve(S)) do
            update VSIDS on resolved literals;
        update VSIDS on learned clause, Backtrack(S);
        if (forget heuristic) then
            Forget(S), Restart(S);
        else
            if (restart heuristic) then
                Restart(S);
    else
        ifrule (! Propagate(S)) then
            Decide(S) literal with max. VSIDS score;
return(S);
```

Implementation: Data Structures

Propagate $(M; N; U; k; \top) \Rightarrow_{\text{CDCL}} (ML^{C\vee L}; N; U; k; \top)$

provided $C \vee L \in (N \cup U)$, $M \models \neg C$, and L is undefined in M

Conflict $(M; N; U; k; \top) \Rightarrow_{\text{CDCL}} (M; N; U; k; D)$

provided $D \in (N \cup U)$ and $M \models \neg D$

Implementation

- data structures: clauses, trail, and the rules
- heuristics: decision literal, forget, restart
- space efficiency: forget
- quality: restarts
- special cases

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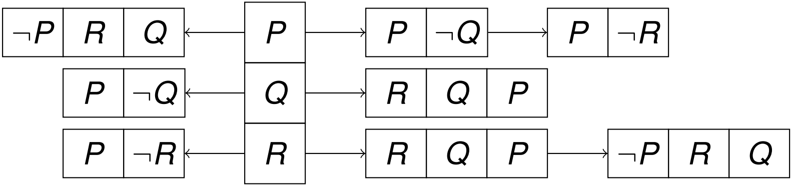
2.10.1 Invariant (2-Watched Literal Indexing)

If one of the watched literals is false and the other watched literal is not true, then all other literals of the clause are false.

$$N = \{P \lor \neg R,\ P \lor \neg Q,\ R \lor Q \lor P,\ \neg P \lor R \lor Q\}$$



VSIDS: Variable State Independent Decaying Sum

- each propositional variable has a positive *score*, initially 0
- decide the variable with maximal score, remember sign (*phase saving*)
- increment the score of variables involved in resolution by b
- increment the score of variables in learned clauses by b
- initially $b > 0$
- at Backtrack set $b := c * b$ where $2 >> c > 1$, i.e., $b_n = c^n * b$
- take care of overflows, i.e., rescale from time to time
- sometimes pick a variable randomly

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Forget

- fix a limit d on the number of learned clauses
- if more than $|U| > d$ start forgetting
- remove redundant clauses
- sort the learned clauses according to a score
- typical elements of the score are clause length, the VSIDS score, dependency on decisions
- remove the $k\%$ clauses with minimal score from U
- $d := d + e$ for some e, $e >> 1$
- do a Restart

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Restart

- after forgetting do a restart
- if a unit is learned do a restart
- restart often at the beginning of a run
- classics: Luby sequence 1, 1, 2, 1, 1, 2, 4, ...
  $(u_1, v_1) := (1, 1)$,
  $(u_{n+1}, v_{n+1}) := ((u_n \& -u_n) = v_n?(u_n + 1, 1) : (u_n, 2 * v_n))$

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Memory Matters: SPASS-SATT

Forget-Start	800	108800
Restarts	412	369
Conflicts	153640	133403
Decisions	184034	159005
Propagations	17770298	15544812
Time	11	23
Memory	16	41

Propositional Logic Calculi

1. Tableau: classics, natural from the semantics
2. Resolution: classics, first-order, prepares for CDCL
3. CDCL: current prime calculus for propositional logic
4. Superposition: first-order, prepares for first-order

Propositional Superposition

Propositional Superposition refines the propositional resolution calculus by

(i) ordering and selection restrictions on inferences,

(ii) an abstract redundancy notion,

(iii) the notion of a partial model, based on the ordering for inference guidance

(iv) a *saturation* concept.

Important: No implicit Condensation of literals!

2.7.1 Definition (Clause Ordering)

Let $\prec$ be a total strict ordering on Σ.

Then $\prec$ can be lifted to a total ordering on literals by $\prec \subseteq \prec_L$ and $P \prec_L \neg P$ and $\neg P \prec_L Q$, $\neg P \prec_L \neg Q$ for all $P \prec Q$.

The ordering $\prec_L$ can be lifted to a total ordering on clauses $\prec_C$ by considering the multiset extension of $\prec_L$ for clauses.