FailBounds

$$(E; B; \beta; S; \top) \Rightarrow_{\text{SIMP}} (E; B; \beta; S; \bot)$$

if there are two contradicting bounds $x \leq c_1$ and $x \geq c_2$ in $B \cup S$ for some variable x

Example:
if $\{x \geq 5, x \leq 0\} \subseteq B \cup S$, then

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EstablishBound

$$(E; B \uplus \{x \circ c\}; \beta; S; \top) \Rightarrow_{\mathsf{SIMP}} (E; B; \beta; S \cup \{x \circ c\}; \mathsf{IV})$$

Example:

$$E := \left\{ \begin{array}{l} u \approx x + 2y, \\ v \approx x - y \end{array} \right\}, \quad \begin{array}{lll} B & := & \{x \geq 0, y \leq -1, u \geq 1, v \geq 2, v \leq 3\} \\ \beta & := & \{x \mapsto 0, y \mapsto 0, u \mapsto 0, v \mapsto 0\} \\ S & := & \{\} \end{array}$$

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$$(E; B; \beta; S; V) \Rightarrow_{\mathsf{SIMP}} (E; B; \beta; S; \top)$$

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$$(E; B; \beta; S; \mathrm{IV}) \Rightarrow_{\mathrm{SIMP}} (E; B; \mathrm{upd}(\beta, x, c, E); S; \mathrm{IV})$$

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if $(x \geq c) \in S$, x dependent, $\mathrm{LRA}(\beta) \not\models x \geq c$, there is an independent variable y and equation $(x \approx ay + t) \in E$ where
($a > 0$ and $\beta(y) < c'$ for all $(y \leq c') \in S$) or
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if $(x \geq c) \in S$, x dependent, $\mathrm{LRA}(\beta) \not\models x \geq c$, there is an independent variable y and equation $(x \approx ay + t) \in E$ where
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FailDepVar$\leq$

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