6.2.5 $E = \{2y + x \geq 1,\ y - x \leq -2,\ x \geq 0\}$

$\uparrow$ simple bound

$E_0 = \{z_1 \simeq 2y + x,\ z_2 \simeq y - x\}$

$B_0 = \{x \geq 0,\ z_1 \geq 1,\ z_2 \leq -2\}$

$\beta_0 = \{x, y, z_1, z_2 \rightarrow 0\}$

$(E_0, B_0, \beta_0, \emptyset, T)$

$\overset{EB}{\Longrightarrow} (E_0, B_0 \setminus \{x \geq 0\}, \beta_0, \{x \geq 0\}, IV)$

$\overset{AB}{\Longrightarrow} (\quad, B_1 \quad, \quad '' \quad, T)$

$\overset{EB}{\Longrightarrow} (E_0, \{z_2 \leq -2\}, \beta_0, \{x \geq 0, z_1 \geq 1\}, IV)$

$\overset{AEB}{\Longrightarrow} (\quad \quad '' \quad, DV)$

$\beta_0(z_1) = 0 \ngeq 1$ nicht

$E_5 = \{x \simeq -2y + z_1,\ z_2 \simeq 3y - z_1\}$

$\beta_5 = \{z_1 \rightarrow 1, y \rightarrow 0, x \rightarrow 1, z_2 \rightarrow -1\}$

$\overset{*}{\Rightarrow} (E5, \cancel{\emptyset}, \beta_5, B_0, \mathrm{D}\vee)$ $\quad B_0 = \{x \geq 0, z_1 \geq 1, z_2 \leq -2\}$

$\beta_5(z_2) = 1 \quad z_2 \leq -2$

$\beta_5(z_1) = 1 \quad z_2 \approx 3y - z_1$ decrease z_2

no bound

picking

$$E_q = \{x \approx -\frac{2}{3}z_2 + \frac{1}{3}z_1, \; y \approx \frac{1}{3}(z_2 + z_1)\} \quad | \cdot 3$$

$$\beta_q = \{z_2 \mapsto -2, \; z_1 \mapsto 1, \; x \mapsto \frac{5}{3}, \; y \mapsto -\frac{1}{3}\}$$

$$\overset{*}{\Rightarrow} (E_q, \emptyset, \beta_q, B_0, \top) \; \checkmark$$

Strict Inequalities

$$1 < 3x + 2y$$

$$E_0 = \{ z_1 \approx 3x + 2y \}$$

$$B_0 = \{ \underline{1 < z_1} \} \qquad B_0 = \{ \underset{\delta > 0}{\underline{1 + \delta}} \leq z_1 \}$$

① one δ suffices

switch from $\mathbb{Q} \rightsquigarrow \mathbb{Q}_\delta = (q, r) \quad q, r \in \mathbb{Q}$

$(q_1, p_1) + (q_2, p_2) = (q_1 + q_2, p_1 + p_2)$

$a(q_1, p_1) = (aq_1, ap_1)$

$(q, r) = q + r \cdot \delta$

$(q_1, r_1) \leq (q_2, r_2)$ iff $(q_1 < q_2)$ or $(q_1 = q_2$ and $r_1 \leq r_2)$

Principle: Do Simple Things First!

Example $(\underbrace{z_1 \geq 1}_{P} \vee \underbrace{3x - y < 2}_{Q})$

FM..

$\wedge\ (z_1 < 1) \wedge (3x - y \geq 2)$

DNF $\rightsquigarrow$ VE (depth exponential) $\rightsquigarrow$ unsat

P

$(P \vee Q) \wedge \neg P \wedge \neg Q$

unsat in prop logic

Boolean Sat $[P, Q, \neg R, \dots]$

test inequations $\rightsquigarrow$ Simplex $Q \wedge \neg R$ ↯

add $\neg Q \vee R$

7.2.3. -3

$$\mathcal{N}' \models_{\tau} at_{\tau}^{-1}(U)$$

Ind on $\Rightarrow^{*}_{CDCL(T)}$

Conflict

<u>Backtrack</u> D $\quad \mathcal{N} \models D \quad \mathcal{N}' \models at_{\tau}^{-1}(D)$

<u>τ-conflict</u> $at_{\tau}^{-1}(L_1), \dots, at_{\tau}^{-1}(L_n) \models_{\tau} \bot$

$\rightsquigarrow \mathcal{N}' \models \neg(at_{\tau}^{-1}(L_1) \wedge \dots \wedge at_{\tau}^{-1}(L_n))$

$$N' = \underbrace{2x \geq 5}_{P} \wedge \left(\underbrace{x \leq 1}_{Q} \vee \underbrace{x \geq 6}_{R} \right)$$
$$\wedge\ \underbrace{x \geq 0}_{S}$$

$$P \wedge (Q \vee R) \wedge S = N$$

$$\overset{*}{\underset{\text{CDCL(T)}}{\Longrightarrow}} \left([P^{P} S^{S} Q^{1}], N, \emptyset, 1, T \right)$$

Simplex Sat?

$$\underbrace{2x \geq 5}_{P} \wedge x \geq 0 \wedge \underline{\underbrace{x \leq 1}_{Q}}$$
$$\underline{x \geq \tfrac{5}{2}}$$

Conflict BC

$$\underset{\text{T-Conflict}}{\Longrightarrow} \quad \neg P \vee \neg Q$$

$$\Longrightarrow \left([P^{P} \neg Q^{\neg P \vee \neg Q} S], N, \{\neg P \vee \neg Q\}, 0, T \right)$$
$$-1, T$$

$$\Longrightarrow ($$

P $\quad \neg Q$

$x \geq \frac{5}{2} \quad x > 1$

S: $x \geq 0$