Prop Logic

$P \vee Q$

$$\frac{P \vee C \qquad \neg P \vee D}{C \vee D}$$

$\models$

FOL Logic

$\forall x, y \; P(x,y) \vee Q(a, g(y))$

$\forall x \exists y \; P(x,y)$

$\models$

LA ← Theory

$\forall x, y \; 3x - 2y < 7$

FOL(LA)

Calculus

Tableau

Resolution

CDCL

CNF

CNF

$\mathbb{Z}$ || Bernays-Schönfinkel

BS

$P(x, a) \quad \neg P(g(y), z)$

FM

Simplex

CDCL(T)

$P(x, g(y), a) \stackrel{?}{=} P(b, z, a)$

$\sigma = \{x \to b, z \to g(y)\}$

$P(b, g(y), a)$

$P(a,$ ~~unifiable~~ $P(b,$

$P(x, -) \qquad P(g(x), -)$

$\sigma = \{x \to g(x)\}$ Occur Check

$P(x, -)\sigma = P(g(x), \ldots)$

$P(g(x), -)\sigma = P(g(g(x)), -)$

$$G = |E| = \{ \overset{1}{f}(\overset{2}{x}, \overset{3}{y}) = \overset{4}{f}(\overset{5}{a}, \overset{6}{z}) \} \qquad E \overset{*}{\underset{SU}{\Longrightarrow}} E' \text{ solution}$$

$$\{x \to a, y \to z\} \quad \{x \to a, z \to y\} \qquad |E| \; ? \; |E'|$$

$$E = \{ f(x, y) = f(z, g(z)) \}$$

$$\{x \to z, y \to g(z)\}$$

$$E = \{ h(x, \underline{y, y}) = h(z, \underline{g(z), z}) \}$$

$$\bot$$

$$E = \{ h(x, y, x) = h(g(y), a, g(b)) \}$$

$$\overset{Dec}{\Longrightarrow} \{ x = g(y), y = a, x = g(b) \}$$

$$\overset{Subst}{\Longrightarrow} \{ x = g(y), y = a, g(y) = g(b) \}$$

$$x \to g(y)$$

$$\overset{Dec}{\Longrightarrow} \{ x = g(y), y = a, y = b \} \overset{Sub}{\Longrightarrow} \{ x = g(a), y = a, a = b \} \Longrightarrow \bot$$

$$\Sigma = \{\, h(x, f(x,x), z, \dots) = h(f(a,a), y, f(y,y), \dots)$$

$$\Sigma' = \{\; x \to f(a,a)$$
$$y \to f(f(a,a), f(a,a))$$
$$z \to f(f(f(a,a), f(a,a)), f(f(a,a), f(a,a)))$$
$$\vdots$$
$$\}$$

Subst Rule

SU

99% > Occur check

$$PU \Rightarrow^{DIC}_{SIM} \{ x = f(a,a),\ y = f(x,x),\ z = f(y,y), \dots \}$$

Subst

no cycles
no merge

Solved form !

Unifier out of PU solved system

$$E = \{x_1 = t_1, \dots, x_n = t_n\}$$

assum! t_i no $x_j \in \mathrm{vars}(t_i)$ $j > i$

if not then cycle

$\sigma = \{x_1 \to t_1\}(\{x_2 \to t_2\}(\dots(\{x_n \to t_n\})\dots))$

const can explain size of initial E (t, σ)

80% → Cycle

no new terms in subterms

$$E = \{\ h(x,y,y) = h(g(z), g(z), z)\ \}$$

$\Rightarrow_{PU}^{Dec}$ $\{\ x = g(z),\ y = g(z),\ \underline{y = z}\ \}$

$\Rightarrow_{PU}^{Subst}$ $\{\ x = g(z),\ \underline{z = g(z)},\ y = z\ \}$

$\Rightarrow_{PU}^{Orient}$

$$E = \{\ P(x,y,g(x)) = P(g(y),a,z)\ \}$$

$\Rightarrow_{PU}^{Dec}$ $\{\ x = g(y),\ y = a,\ g(x) = z\ \}$

$\Rightarrow_{PU}^{Elim}$ $\{\ x = g(y),\ \underline{y = a},\ \underline{z = g(x)}\ \}$

No Cycle!

$$t\ \{z \to g(x)\}\{x \to g(y)\}\{y \to a\}$$

$$\sigma = \{z \to g(g(a)),\ x \to g(a),\ y \to a\}$$