

$$\exists x \forall y, z (R(x, y) \rightarrow R(x, z))$$

for constants

iff
SAT

$$\forall z, y (R(a, y) \rightarrow R(a, z))$$

$$\neg R(a, y) \vee R(a, z)$$

BS clause set N, B are all constants

$\text{Gnd}(N)_B$ is an grounding of N over B

N is SAT iff $\text{Gnd}(N)_B$ is SAT
 $\neg R(a, y) \vee R(a, z)$ iff $\neg R(a, a) \vee R(a, a) \leftarrow \text{SAT}$

$$T = [R(a, b)^2]$$

$$\left(\underbrace{\neg R(x, y) \vee \neg R(x, z)} \vee R(y, z) \right) \{x \rightarrow a, y \rightarrow b, z \rightarrow b\}$$

$$\neg R(a, b) \vee \neg R(a, b) \vee R(b, b)$$

$$\neg R(a, b) \vee \underline{R(b, b)}$$

$$\neg R(x, z) \vee \underbrace{\neg R(x, z)} \vee R(z, z)$$

$$\neg R(x, z) \vee R(z, z)$$

$$\Rightarrow \text{Project} \left[R(a, b)^2 \wedge R(b, b) (\neg R(x, z) \vee R(z, z)) \cdot \{x \rightarrow a, z \rightarrow b\} \right]$$

$$\begin{array}{ccc}
 R(x, y) \vee R(z, z') & & R \\
 \neg R(x', y') \vee \neg R(z', z'') & \rightsquigarrow & \neg R
 \end{array}$$

2 literals

$$\begin{array}{l}
 \downarrow \\
 R(x, y) \text{ by } \{z \rightarrow x, z' \rightarrow y\} \\
 \neg R(x', y') \text{ by } \{z' \rightarrow x', z'' \rightarrow y'\}
 \end{array}$$

$$N = \left\{ \begin{array}{l} \underline{R(a,b)} \vee \underline{R(a,c)} \\ \underline{\neg R(x,y)} \vee \underline{R(y,x)} \\ \neg R(b,a) \end{array} \right\} \quad B = \{a, b, c\}$$

Promysel

$$\Rightarrow ([\underline{\neg R(b,a)} \neg R(b,a)], N, \emptyset, 0, T)$$

$$\{y \rightarrow b, x \rightarrow a\}$$

Promysel

$$\Rightarrow ([\neg R(b,a) \neg R(a,b) \neg R(x,y) \vee R(y,x) \{y \rightarrow b, x \rightarrow a\}], \neg)$$

Promysel

$$\Rightarrow ([\neg R(b,a) \neg R(a,b) \overline{R(a,c)} \overline{R(b,b)} \vee R(a,c)], \neg)$$

Dec

$$\Rightarrow ([\underline{\underline{R(c,c)}}, \underline{R(a,c)}], \dots)$$

$$N = \left\{ \begin{array}{l} \neg R(x, a) \vee \neg R(x, b) \\ R(a, b) \vee R(b, a) \end{array} \right\} \quad B = \{a, b\}$$

Decide

$$\Rightarrow ([R(a, b)]^2, N, \emptyset, \wedge, \neg, \top)$$

$$(\neg R(x, a) \vee \neg R(x, b)) \left\{ \begin{array}{l} x \rightarrow a \\ y \rightarrow b \end{array} \right\}$$

Conflict

$$\Rightarrow ([\neg R(a, b)]^2, N, \emptyset, \wedge, (\neg R(x, a) \vee \neg R(x, b)))$$

Nothing to solve

$$([R(b, a)]^2)$$

It is forbidden to do a Decide leading to a Conflict

$$\mathcal{A} = \begin{cases} \underline{R(a,b)} \cup R(a,a) \\ \neg Q(x,y) \vee \neg Q(x,z) \\ \neg R(x,y) \vee Q(x,y) \end{cases}$$

$$B = \{a, b\}$$

$$\Rightarrow \text{Slide} \left(([R(a,a)]^2, N, \emptyset, \perp, \top) \right. \\ \left. \left(\neg R(x',y') \vee Q(x',y') \right) \{x \rightarrow a, y \rightarrow a\} \right)$$

$$\Rightarrow \text{Pop} \left([R(a,a)]^2, \underline{Q(a,a)} \right), N, \emptyset, \perp, \top$$

$$\Rightarrow \text{Case} \left(\begin{array}{c} \neg Q(x,y) \vee \neg Q(x,z) \\ \downarrow \\ \{ \begin{array}{l} x \rightarrow a \\ y \rightarrow a \\ z \rightarrow a \end{array} \} \end{array} \right)$$

$$\Rightarrow \text{Fac} \left(\begin{array}{c} \neg Q(x,z) \\ \downarrow \\ \{ \begin{array}{l} y \rightarrow z \\ x \rightarrow a, z \rightarrow a \end{array} \} \end{array} \right)$$

$$\Rightarrow \text{Res} \left([R(a,a)]^2, N, \emptyset, \perp, \neg R(x,z) \cdot \{x \rightarrow a, z \rightarrow a\} \right) \\ \Rightarrow \text{Res} \left([R(a,a)]^2, N, \emptyset, \perp, \underline{\neg R(x,z)} \right) \mathcal{F} = \{x' \rightarrow x, y' \rightarrow z\}$$

$$\uparrow R(a, a, a) \quad \Sigma = \{a, b\}$$

$$\begin{aligned}
 &\neg R(x, y, a) \vee R(x, y, b) \quad \text{Counter} \\
 &\neg R(x, a, b) \vee R(x, b, a) \quad \neg R(x, a, a) \vee R(x, b, a) \\
 &\neg R(a, b, b) \vee R(b, a, a) \\
 &\neg R(b, b, b)
 \end{aligned}$$

$$\Rightarrow R_{i,p}^* (R(a, a, a), R(a, a, b), R(a, b, a), R(a, b, b))$$

$$R(b, a, a), R(b, a, b), R(b, b, a)$$

$\Delta \rightarrow R(b, b, b)$ from exp. steps
 exp. resolution steps