

$$l \approx r$$

(i)  $l$  not a variable

$$l \rightarrow r, \quad x \rightarrow t \quad \leftarrow U = []$$

$$f(a) \xrightarrow{x \mapsto a} \mathbb{R} \quad f(t) \xrightarrow{x \mapsto f(t)} t$$

(ii)  $\text{var}(t) \subseteq \text{var}(e)$

$$\underline{f(x)} \rightarrow \underline{g(y)}$$

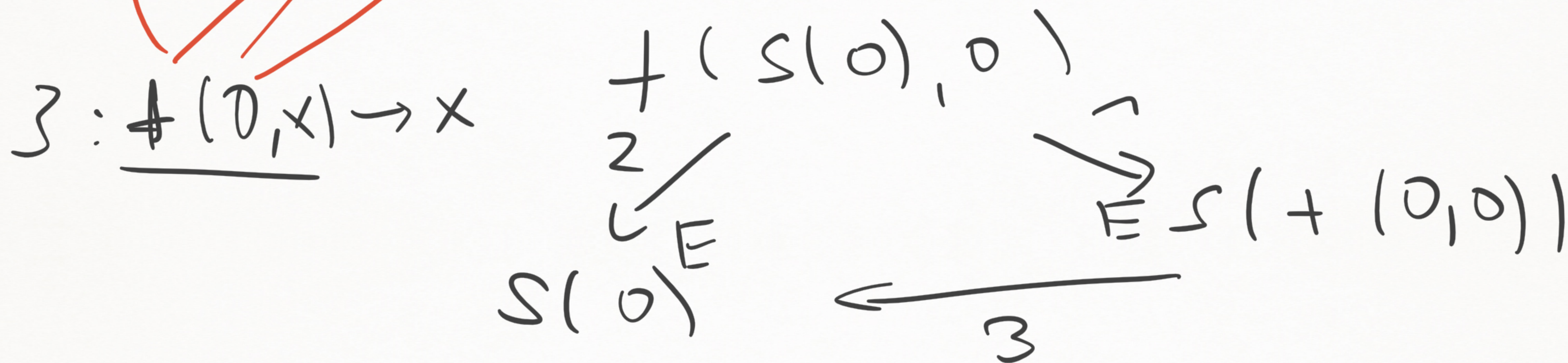
$$f(x) \rightarrow x$$

$$f(g(x)) \rightarrow g(f(x))$$



$$\mathbb{F} = \{ \alpha + (s(x), y) = s + (x, y) \},$$

$$\{ \alpha + (s(0), 0) = s(0) \}$$



$K \cap D \subseteq \alpha = \underline{1} \quad + > S \quad \text{Converged system}$

$$\alpha + (s(0), 0) \xrightarrow{\alpha} s + (0, 0) \xrightarrow{\alpha} s(0)$$

$\alpha + 3 \text{ convergent + reduced} \quad S(0) = S(b) \text{ trivial deduce } K$

Proof:  $s \xrightarrow[\underline{E}]{*} t \iff \exists E \xrightarrow[\underline{E}]{*} s \approx t$

$\Rightarrow$ : ind length  $\leftarrow i$

$i=1$  (i)  $s \leftrightarrow s$

(ii)  $s \leftrightarrow t$  for  $s \approx t \in \underline{E}$

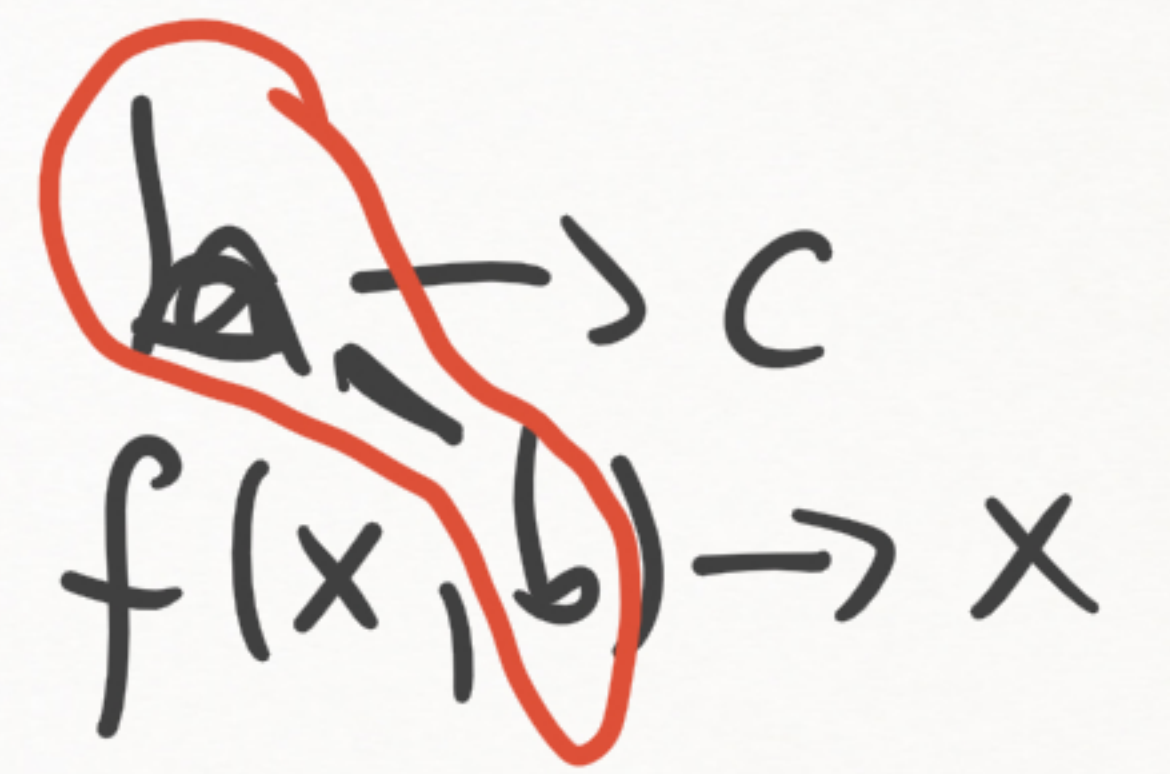
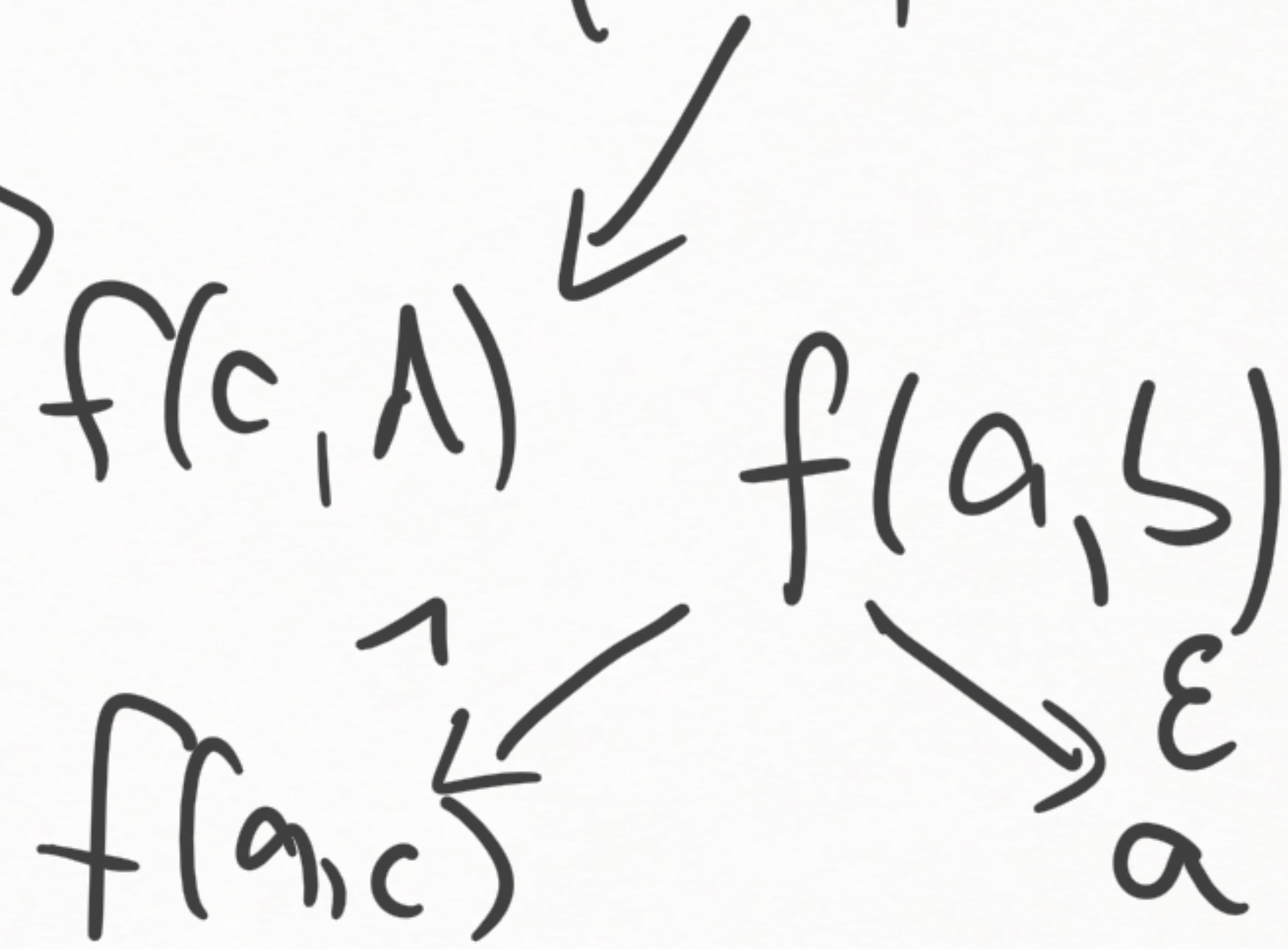
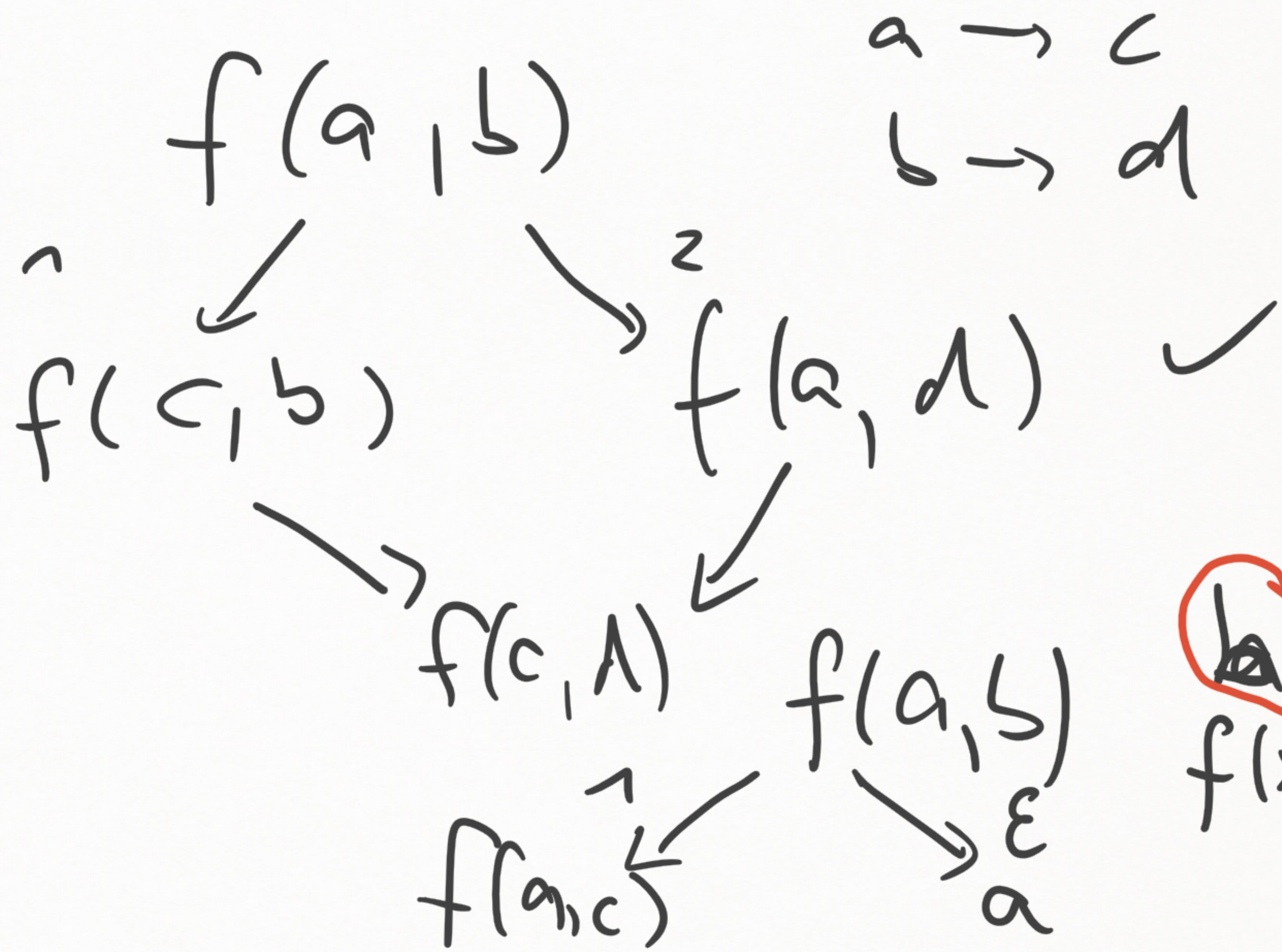
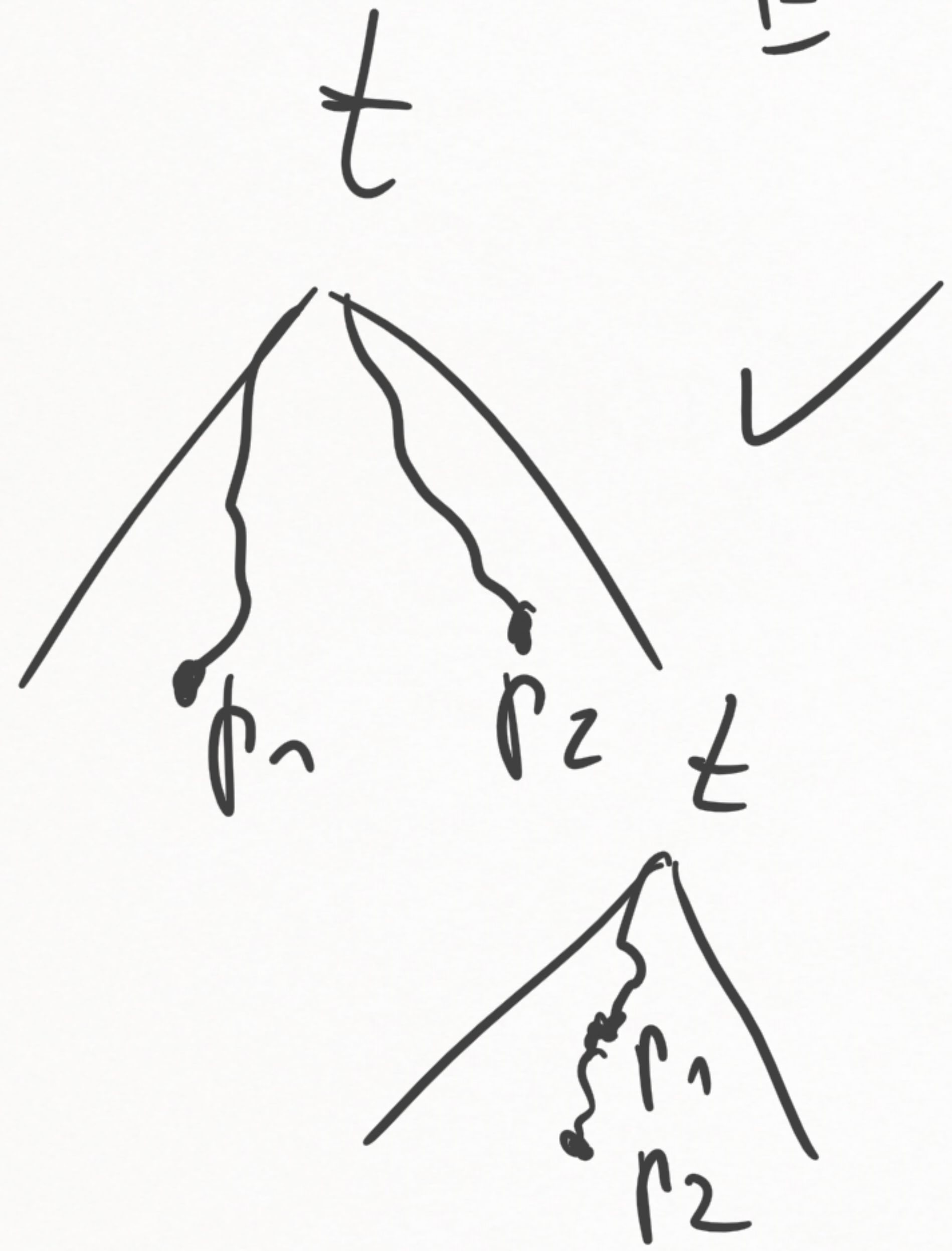
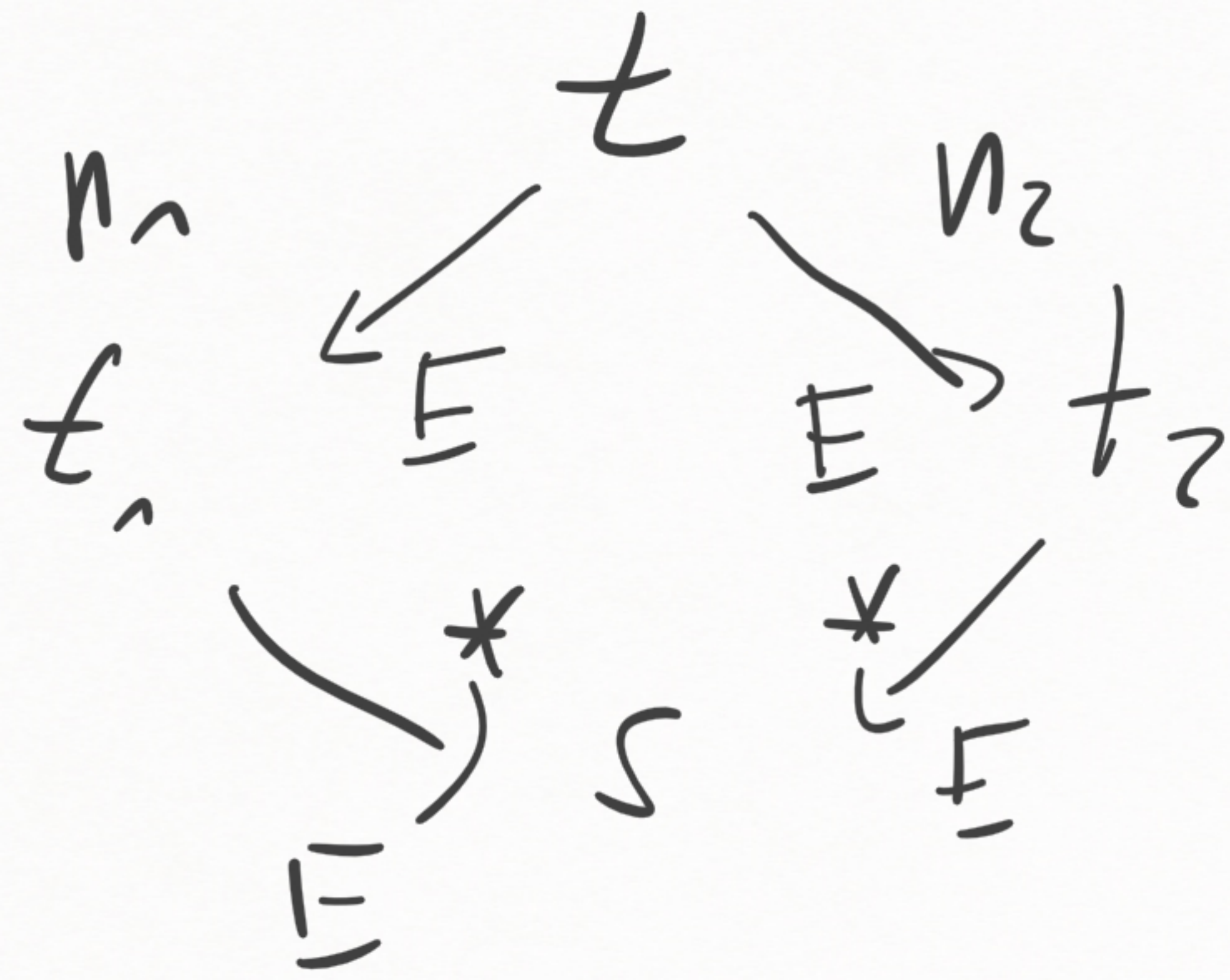
(iii)  $s \rightarrow t$   $s_p = e \sigma, e \approx v \in E, t = s[\sigma]$

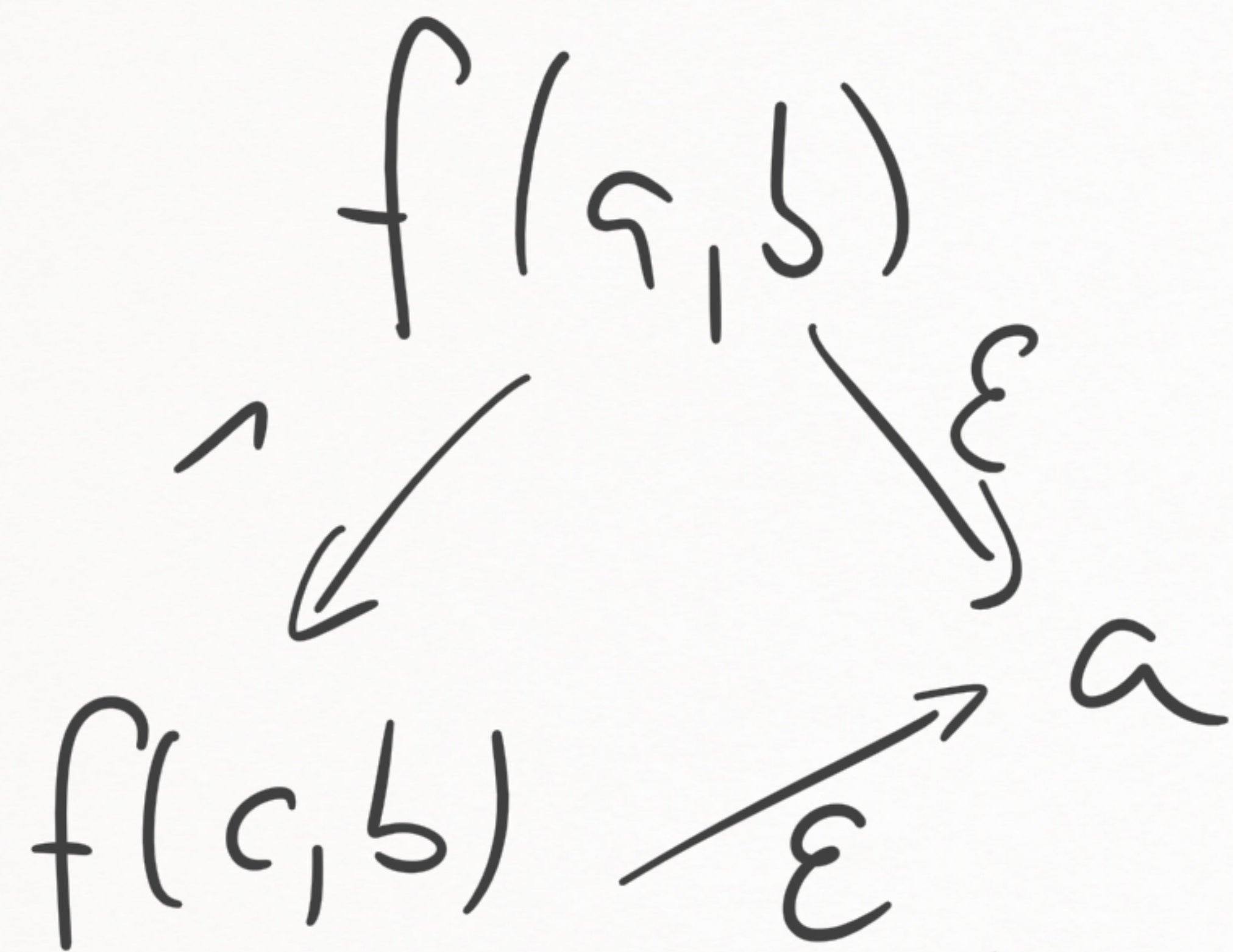
(i)  $\xrightarrow[\text{Ref}]{*} s \approx s \checkmark$

(ii)  $\xrightarrow[\text{Ind}]{*} s \approx t$  for matrix  $E \checkmark \quad \sigma = \{ \}$

(iii) ind depth of position  $p$

$p = \varepsilon$   
 $e \sigma \rightarrow v \sigma \rightsquigarrow \xrightarrow[\text{Ind}]{*} e \approx v \checkmark$   
 + apply Ref + Congruence





$$a \rightarrow c$$

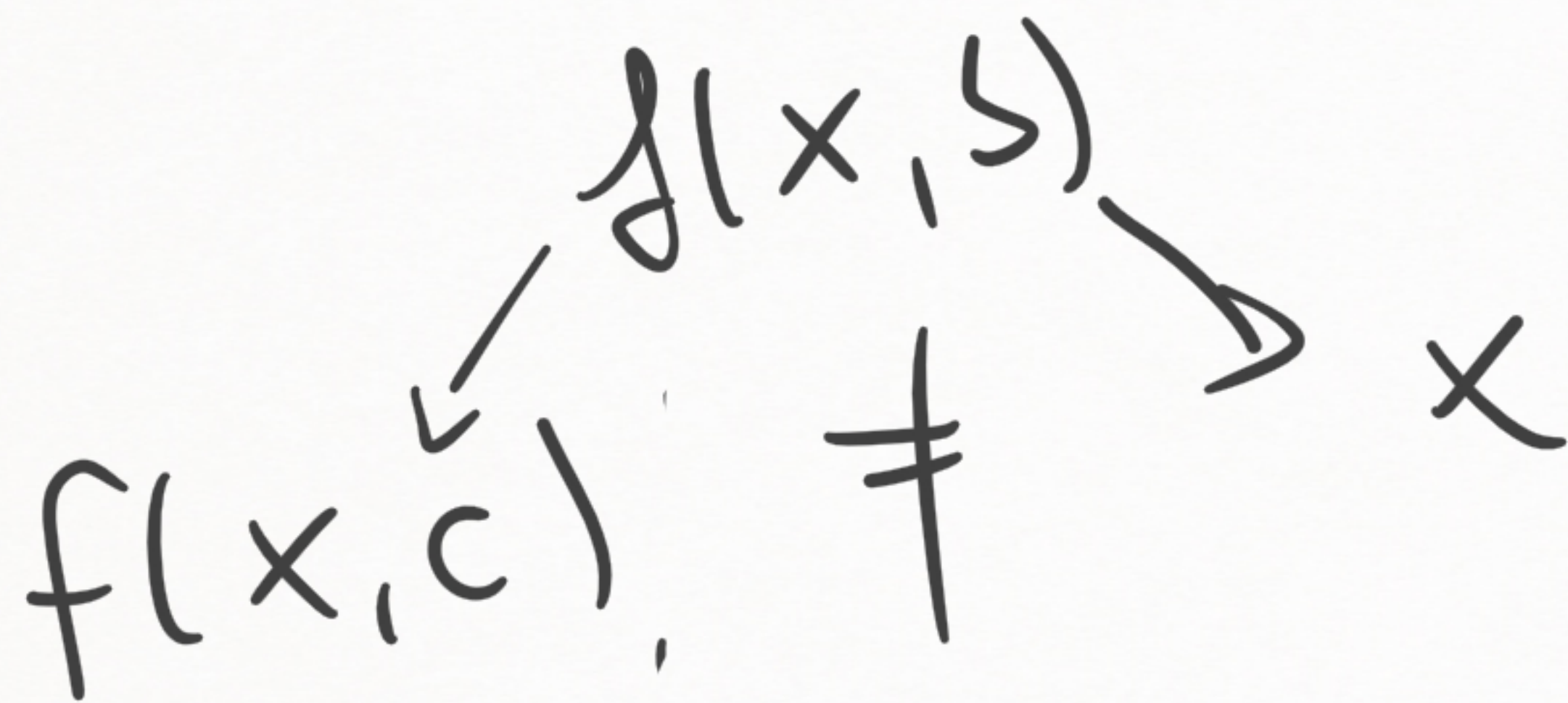
$$f(x, b) \rightarrow a$$

$$R = \{ f(x, b) \rightarrow x, \underline{b} \rightarrow c \} \quad \neq \emptyset \quad b > c$$

$$r = 2 \quad A = \{ \}$$

$$\leadsto R_n = R \cup \{ \underline{f(x, c)} \rightarrow x \}$$

Critical pair.



$$\underline{\underline{f(x, c) \xrightarrow[\varepsilon]{*} x}}$$

$R_n$  convergent

Decidability / Equation

$f(x, y) \rightarrow \begin{matrix} C \\ x \end{matrix}$  decidable

$S \approx t$  follows

$f(x, x) \rightarrow \begin{matrix} C \\ x \end{matrix}$  undecidable

==

BS

$R(x, x)$

$\neg (Q(x, y, x) \vee Q(a, x, y))$  decidable

Axiomatize Equality:

~~$E(x, x)$~~

~~$\neg E(x, y) \vee E(y, x)$~~   $\square$

~~$\neg E(x, y) \vee \neg E(y, z) \vee E(x, z)$~~

$\neg E(x, y) \vee E(g(x), g(y))$

$\neg E(x, y) \vee \neg P(x) \vee P(y)$

• BS fragment

$\neg E(x_1, x_2) \vee \neg E(y_1, y_2) \vee E(f(x_1, y_1), f(x_2, y_2))$

• BS + Equality

• decidability  
linear

$E(t, s)$

decidable



$$\mathcal{R} = \left\{ \begin{array}{l} f(f(x,x), y) \rightarrow g(x) \\ f(z, y') \rightarrow z \end{array} \right\}$$

3 (curved arrow from top  $f$  to  $f(x,x)$ )  
1 (vertical arrow from top  $f$  to  $f(z, y')$ )  
2 (diagonal arrow from top  $f$  to  $f(x,x)$ )

3.  $f(x, x) \quad f(f(x', x'), y')$

$x \rightarrow f(x', x')$   
 $y' \rightarrow f(x', x')$   
 $f(f(f(x', x'), f(x', x')), y')$

$f(g(f(x', x')), y')$      $g(f(x', x'))$

Critical Pairs:

1  $f(z, y') \quad f(f(x, x), y) \quad z \rightarrow f(x, x) \quad y' \rightarrow z$

$f(f(x, x), y) \rightarrow g(x)$   
 $\rightarrow f(x, x)$

2  $f(z, y') \quad f(x, x) \quad z \rightarrow x \quad y' \rightarrow x$

$f(f(x, x), y) \rightarrow g(x)$   
 $\rightarrow f(x, y)$