

$$E = \{ g(a) = b, f(g(a)) = c, \underline{f(c) = g(a)} \}$$

$\rightarrow f(f(g(a))) \neq c$ KBC \uparrow
KBO $f > g > c > b > a$ convergent system
 $w = 1$

\Rightarrow^* Orient
KBC $(\emptyset, \{ \underline{g(a) \rightarrow b}, \underline{f(g(a)) \rightarrow c}, \underline{f(c) \rightarrow g(a)} \})$

\Rightarrow^2 Simplify
KBC $(\{ f(b) = c \}, \{ g(a) \rightarrow b, f(c) \rightarrow b \})$
 \Rightarrow Orient $(\emptyset, \{ f(b) \rightarrow c, g(a) \rightarrow b, f(c) \rightarrow b \})$

$f(f(g(a))) \xrightarrow{g(a) \rightarrow b} f(f(b)) \xrightarrow{f(b) \rightarrow c} f(c) \xrightarrow{f(c) \rightarrow b} b$
 $b \neq c$ from convergent f $f(c) \rightarrow b$ b
 $f(f(g(a))) \neq c$ val 1

$$E = \{ \underline{g(a)} \approx b, \underline{f(g(a))} \approx c, \underline{f(c)} = \underline{g(a)} \}$$

⇒ Flattening. 1.1, 2.1.1, 3.2 fresh d

$$E = \{ d \approx b, f(d) \approx c, f(c) \approx d, g(a) \approx d \} \checkmark$$

$$(\{ d \approx b \}, \{ \underline{f(d)} \rightarrow c, f(c) \rightarrow d, g(a) \rightarrow d \})$$

$$\xRightarrow[\text{cc}]{\text{Omit}} (\emptyset, \{ \underline{b \rightarrow d}, f(d) \rightarrow c, f(c) \rightarrow d, g(a) \rightarrow d \}) / \underline{\text{down}}$$

$$d \rightarrow b \quad \uparrow$$

Innermost
Reduction

$$f(f(g(c))) \neq c$$

$$g(a) \rightarrow d \rightarrow f(\underline{f(d)}) \xrightarrow{f(d) \rightarrow c} f(c) \rightarrow d \xrightarrow{d \rightarrow b} b$$

Innermost Rewriting

$$g(a) \rightarrow \underline{b}$$

$$f(g(a)) \rightarrow g(c)$$

$$f(g(c)) \rightarrow d$$

Outermost

$$\underline{f(f(g(a)))}$$

Innermost

$$d \leftarrow f(g(c))$$

$$f(f(b))$$

Congruence Class Properties

→ Soundness + Completeness by KBC
+ invariants (i) $E = \{ c_i \approx c_j \}$
(ii) $R = \{ c_i \rightarrow c_j, f(c_1, \dots, c_n) \rightarrow c \}$

→ Termination?

lex ordering of term in $E \cup R$
plus size of E

Example:

not valid

Valid: $\forall x, y [f(f(x)) \neq x \vee f(x) \neq y \vee f(f(y)) \neq g(y) \vee x = y \vee h(x, y) = h(x, g(y))]$

Unsat: $\exists x, y [f(f(x)) = x \wedge f(x) = y \wedge f(f(y)) = g(y) \wedge x \neq y \wedge h(x, y) \neq h(x, g(y))]$

$x \rightarrow a$
 $y \rightarrow b$: $f(f(a)) = a \wedge f(a) = b \wedge f(f(b)) = g(b) \wedge a \neq b \wedge h(a, b) \neq h(a, g(b))$

$E = f(f(a)) = a \wedge f(a) = b \wedge f(f(b)) = g(b)$

Sat

Flattening $f(a)$: 1.1.1, 2.1, 3.1.1
 $E = [f(c) = a \wedge c = b \wedge f(c) = g(b) \wedge f(a) = c]$

Flattens $g(b)$: 3.2 (3.1 not a constant)
 $E = [f(c) = a, c = b, f(c) = d, f(a) = c, g(b) = d]$

$(\{c = b\}, \{f(c) \rightarrow a, f(c) \rightarrow d, f(a) \rightarrow c, g(b) \rightarrow d\})$

$\Rightarrow (\{c = b, a = d\}, \{f(c) \rightarrow d, f(c) \rightarrow c, g(b) \rightarrow d\})$
 $\Rightarrow (\{c = b\}, \{f(c) \rightarrow d, f(a) \rightarrow c, g(b) \rightarrow d, d \rightarrow a\}) \Rightarrow (\emptyset, \{f(c) \rightarrow d, f(a) \rightarrow c, g(b) \rightarrow d, c \rightarrow b\})$

$(\emptyset, \{f(c) \rightarrow d, f(a) \rightarrow c, d \rightarrow a, c \rightarrow b, g(b) \rightarrow d\})$ $d \rightarrow a$

Inequations: $a \neq b \times h(a, b) \neq h(a, g(b))$ $c > b$

