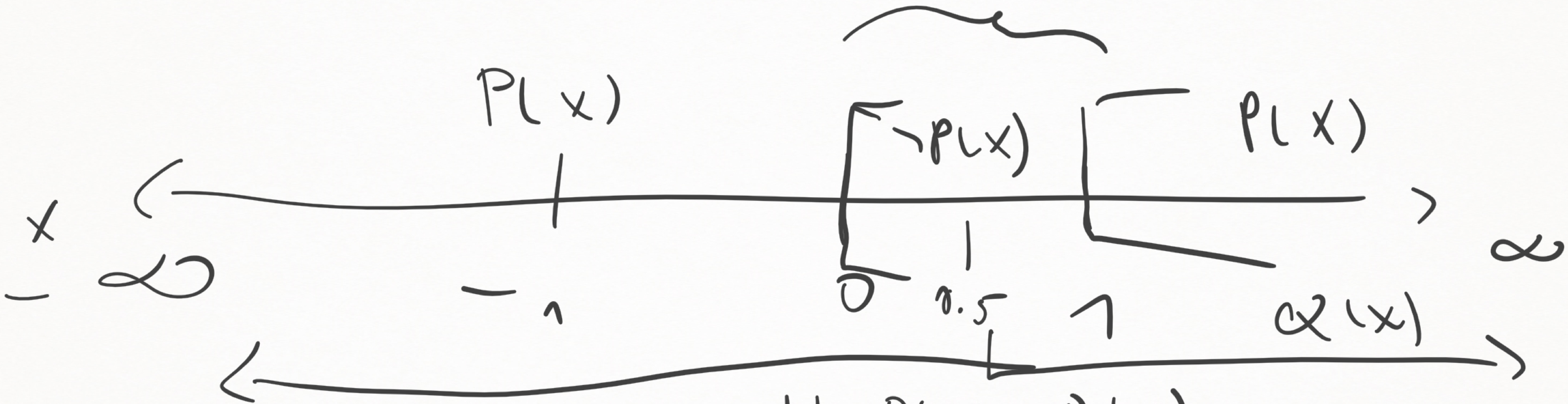
$$X = 0 \;\|\; P(x)$$

$$y = x + 1 \;\|\; \neg P(x) \vee P(y)$$

$$=\!=\qquad B = \{a, b, c\}$$

$$\xRightarrow[\sigma = \{x \to a\}]{\text{Propagate}} \big([\, P(a),\ a = 0 \,], \ldots \big)$$

$$\xRightarrow[\sigma = \{x \to a,\ y \to b\}]{\text{Propagate}} \big([\, P(a),\ a = 0,\ P(b),\ b = 1 \,], \ldots \big)$$



$2x \geq 1 \quad || \quad P(x) \vee Q(x)$

$x \geq \frac{1}{2}$

FOL (LRA) : Test Point Method

$$\forall \; \exists x, y, t \; \left(\left(\underbrace{x \geq 3y + 1}_{} \lor z < y \right) \land 2x + y \leq z \right)$$

$x \leq \frac{z - y}{2}$

Eliminate x

iff $\bigvee$

$x = 3y + 1$

$x = \infty$

$x = \frac{z - y}{2}$

$x = -\infty$

$x \to 3y + 1$

$x = t$

$x \leq t \land x \geq t$

$x < 3y + z$ $\quad \varepsilon > 0$

$x = 3y + z + \varepsilon$

$\Delta \| \neg P(x) \lor P(y)$

BS (SB)   SB = simple Bounds

LRA atoms :

$x \geq$ const
$x \leq$ const
$x =$ const
$x <$ const
$x >$ const

Simple Bounds

$x > 5 ,\ y < 7 \ ||\ R(x,y)$

+ | no overlaps between variable Bounds or segments |

B

$-\infty \quad 5 \sim 7 \quad +\infty$

$a_0 < a_1 < a_2 < a_3 < a_4 < a_5 < a_6$

$x < 5 \ ||\ \neg R(x,y)$

=

NRA $\quad x^2 < 5 \ ||\ P(x)$

Proof

$\xRightarrow{\text{Conflict}} (M'' ; N ; U ; B ; k ; \Delta_0 \,\|\, C_0 \cdot \sigma_0)$ ← one lit of level k

$\Downarrow$ Skip, Fact, Resolve

$\Rightarrow^* \quad (M, K^{i+1}, M' ; N ; U ; B ; k ; \Delta_n \,\|\, C_n \cdot \sigma_n)$

$\xRightarrow{\text{Backtrack}} (M, L\sigma^{(\Delta_n \| D \vee L)\sigma}, \Delta_n\sigma ; N ; U \cup \{\Delta_n \| C_n\} ; B ; i ; \top)$

$\sigma_n = \sigma$, $C_n = D \vee L$

$(\Delta_n \| D \vee L)\sigma$ is non-red. < $M = [L'_1, L'_2, \ldots, L'_n]$

$L'_1 < L'_2 < \ldots < L'_n$

$(N \cup U) \models \Delta_n \| C_n$ $\quad \Delta_n\sigma \wedge M \wedge \mathrm{adiff}(B)$ satisfiable

$C_n\sigma$ is false under M and M, K^{i+1}, M'.

By contradiction. $N' \subseteq \mathrm{gnd}((\Sigma, B, T), (N \cup U))$

such that $N' \models \Delta_n\sigma \| C_n\sigma$ ← is false

$\leq \Delta\sigma \| C_n\sigma$

$\Delta' \| C' \in N'$ and false Δ' ist sat in $M \wedge \mathrm{adiff}(B)$

highest L from M'' is not in C' ↯ regular run

On BS (LRA) resolution
is complete on ground clause sets.

Clause set unsat
$\Rightarrow$ resolution refutation

$\Rightarrow$ complete grounding
$\hookrightarrow$ hierarchic Herbrand Theorem