1) Factory vs. Coalgebra (Superposition)

- Coalgebra is a simplification you always want to do that
- Factory adds claim: restrict interface

2) First-Order: Quantifier over Vars
Second-Order: + Quantification over Relations, Functions

$$\forall P : \left[P(0) \wedge \forall x \left(P(x) \rightarrow P(x+1)\right)\right] \rightarrow \forall y\, P(y)$$

3) Risk + Detailed Analysis & Verification
Hardware, Software with Risk, Product Modelling

$\forall \vec{x}\ \Phi$ valid ¬valid

$\longleftrightarrow \neg \forall \vec{x}\ \Phi$ unsat

$\exists \vec{x}\ \neg \Phi \rightsquigarrow x_i \longrightarrow c_i$

$\rightsquigarrow$

unsat

sat

$\exists x$

$x \geq 6 \wedge (x < 5 \vee y > 9)$

$c \geq 6 \wedge (c < 5 \vee d > 9)$

$A \wedge (B \vee D)$

$[A, B^1]$

Lythm $c \geq 6 \wedge c < 5$ N/

$\neg A \vee \neg B$

$[A, \neg B^{\neg A \vee \neg B}, D^{D \vee D}]$ $c \geq 6 \wedge c \geq 5 \wedge d > 9$

$$y < 5 + \frac{1}{2}(z+3) = \frac{13}{2} + \frac{1}{2}z$$

$$y \leq \frac{3}{2}(z+3) + 2 - z = \frac{1}{2}z + \frac{13}{2}$$

$$y - 11 + \frac{3}{2}(z+3) \geq z$$

FM(y)

$$y \geq 11 - \frac{3}{2}z - \frac{9}{2} + z = \frac{13}{2} - \frac{1}{2}z$$

$$\vdots$$

$$\frac{13}{2} + \frac{1}{2}z > \frac{13}{2} - \frac{1}{2}z$$

$$z > 0$$ done

11.7. a)

$C \vee P(t) \in N$

$\rightsquigarrow$ all models are satisfiable for N

you can delete $C \vee P(t)$

Blocked clause elimination

Proof by contradiction

~~$A \vee B$~~ $\rightarrow$ $A \vee \neg A$

$\neg B \vee \neg A$

$B \vee D$

$\rightsquigarrow$ Model $\mathcal{A}$ for $N \setminus \{C \vee P(t)\}$

no Model for $N \cup \{C \vee P(t)\}$

$\mathcal{A} \not\models (C \vee P(t))\sigma$

$\mathcal{A} \models D\sigma \in N$

$\mathcal{A} \models C\sigma$

$\| (C \vee C'_1)\sigma$

$((\vee C'_n)\sigma$ but $\rightsquigarrow \mathcal{A} \not\models C'_i\sigma$

for a ground instance

$P(s)\sigma = P(t)\sigma$

$\mathcal{A} \not\models P(t)\sigma$

$\mathcal{A} \models \neg P(s)\sigma$

$\| (\neg P(s_1) \vee C'_1)\sigma$

$(\neg P(s_k) \vee C'_k)\sigma$

$\rightsquigarrow \mathcal{A}' = \mathcal{A}$ but $P(t)\sigma$ true

false ↯ ⟶ [

	N_c	δ_c
$\neg R(a,a)^*$	$\emptyset$	$\emptyset$
$\neg R(a,a)^* \vee R(a,a)$	$\emptyset$	$\emptyset$
$R(a,g(a))^*$	$\emptyset$	$\{R(a,g(a))\}$
$R(g(a),a)^* \vee \neg R(a,g(a))$	$\{R(a,g(a))\}$	$\{R(g(a),a)\}$
$\neg R(g(a),a)^*$	$\{R(a,g(a)), R(g(a),a)\}$	$\emptyset$
$\neg R(g(a),a)^* \vee R(a,g(a))$	"	$\emptyset$

$$(\{ f(g(x),y) = f(x,y) \}, \{ f(g(a),a) \xrightarrow{2} f(b,a),\ g(g(x')) \xrightarrow{1} g(x') \})$$

Orient

$$\Rightarrow (\emptyset, \{ f(g(x),y) \xrightarrow{3} f(x,y), \text{[illegible]} \})$$

$$x \to g(x')$$

$$\Rightarrow (\{ f(g(x'),y) = f(g(x'),y) \}, \})$$

$$f(g(g(x')),y) \xrightarrow{3} f(g(x'),y), \quad \xrightarrow{2} f(g(x'),y)$$

Delete

$$\Rightarrow (\emptyset, \{ \_\_\_ \})$$

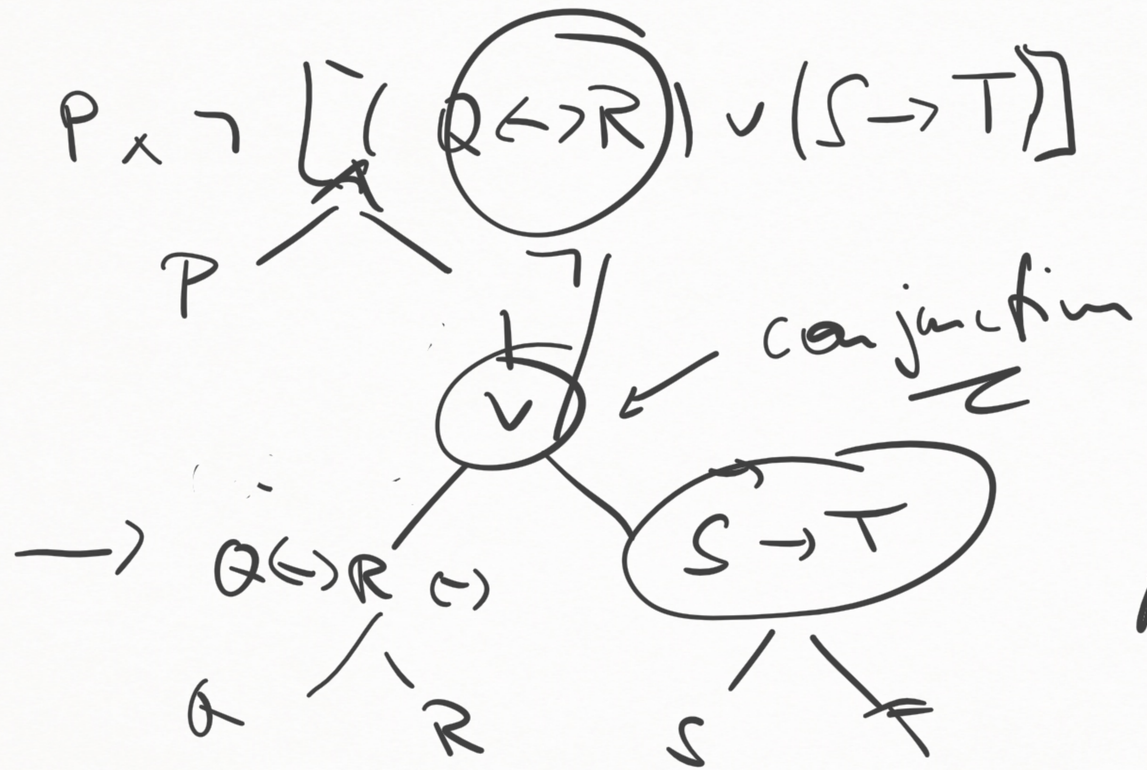
$$\Rightarrow (\{ f(b,a) = f(a,c) \}, \{ \_\_3\_\_ \})$$

Ori

$$\Rightarrow (\emptyset, \{ f(b,a) \to f(a,a),\ f(g(x),y) \to f(x,y),\ f(g(a),a) \to f(b,a),\ g(g(x)) \to g(x) \})$$

Done

11.4



No Renaming
No Skolemization

$\Rightarrow_{\text{AcNF}}$ $P \wedge \neg [((Q \wedge R) \vee (\neg Q \wedge \neg R)) \vee (S \rightarrow T)]$

$\Rightarrow_{\text{AcNF}}$ $P \wedge \neg [(\quad \| \quad) \vee (\neg S \vee T)]$

$\Rightarrow^{*}_{\text{AcNF}}$ $P \wedge [[(\neg Q \vee \neg R) \wedge (Q \vee R)] \wedge S \wedge \neg T]$ ✓

11.5 $f(a, g(a)) \approx f(b, g(b))$, $g(a) \approx h(c)$, $h(d) \approx g(b)$

$d \approx c$, $f(a, h(d)) \approx f(h(d), a)$ $\quad d > c$

$f(a, g(a)) \approx f(b, g(b))$, $g(a) \approx h(c)$, $h(c) \approx g(b)$, $f(a, h(c)) \approx f(h(c), a)$

$\Rightarrow$ $g(a) \approx e_1$, $g(b) \approx e_2$, $h(c) \approx e_3$, $f(b, e_2) \approx e_4$, $f(e_3, a) \approx e_5$

$f(a, e_1) \approx f(b, e_2)$, $e_1 \approx e_3$, $e_2 \approx e_3$, $f(a, e_3) \approx f(e_3, a)$

$f(a, e_1) \approx e_4$, $\qquad f(a, e_3) \approx e_5$

$\overset{*}{\Rightarrow}$ ({ $g(a) \approx e_3$, $g(b) \approx e_3$, $h(c) \approx e_3$, $f(b, e_3) \approx e_4$, $f(e_3, a) \approx e_5$, $f(a, e_3) \approx e_4$,
$f(a, e_3) \approx e_5$ }, { $e_1 \to e_3$, $e_2 \to e_3$ })

$\Rightarrow^*$ ({ $g(1) \to e_3$, ... }]

$$f(l, s(l)) \neq f(h(c), a)$$

$$f(b, e_3) \neq f(e_3, a)$$

$$e_4 \neq e_5$$

true