Exam:

1) Tableau
2) CDCL
3) CNF
4) Sup Model Building
5) Sup or Res Refutation
6) Proof

1 – 5
Calculation

90 min

December 2.
4^{15} pm

$P < Q < R < S$

$\ell \qquad\qquad r$

$P \lor \underline{Q} < P \lor \underline{R} \qquad Q < R$

$P \lor P \lor \underline{\underline{Q}} < P \lor Q \lor \underline{\underline{Q}}$

$P \lor \neg \underline{Q} > P \lor P \lor Q \lor \underline{Q} \qquad Q < \neg Q$

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1) $P \lor R^{*}$
2) $\neg P \lor Q^{*}$ ← Smallest.
3) $Q \lor S^{*}$
4) $Q \lor \neg R^{*}$

$\wedge$

$\neg P \lor Q$
$P \lor R$
$Q \lor \neg R$
$Q \lor S$

$P < Q < R < S$

N	N_D	δ_D	why?
$\neg P \vee Q^*$	$\emptyset$	$\emptyset$	$P \notin N_D$, clause is true
$P \vee R^*$	$\emptyset$	$\{R\}$	$P \notin N_D$
$Q \vee \neg R^*$	$\{R\}$	$\emptyset$	$\neg R$ negative
$Q \vee S^*$	$\{R\}$	$\{S\}$	$Q \notin N_D$

$N_I = \{\underline{\underline{R}}, S\}$ $\quad N_I \not\models N$

$N_I \not\models Q \vee \neg R^*$ $\quad P \vee R^*$

$\Rightarrow_{sup}$ $\quad Q \vee P$

	N_D	S_D	Comment
$P \vee Q^*$	$\emptyset$	$\{Q\}$	$P \notin N_D$
$\neg P \vee Q^*$	$\{Q\}$	$\emptyset$	true
$P \vee R^*$	$\{Q\}$	$\{R\}$	$P \notin N_D$
$Q \vee \neg R^*$	$\{Q, R\}$	$\emptyset$	true
$Q \vee S^*$	$\{Q, R\}$	$\emptyset$	true

$P < Q < R < S$

$$N_I = \{Q, R\}$$

$$N_I \models N \quad \text{saturated}$$

$P < Q < R < S$

1 $Q \vee R \vee R$

2 $\neg P \vee S$

3 $\neg P^{+} \vee Q$

4 $R \vee S$

	N_D	δ_D	
$\neg P^{+} \vee Q^{*}$	$\emptyset$	$\emptyset$	true, $\neg P$ selected
$Q \vee R^{*} \vee R^{*}$	$\emptyset$	$\emptyset$	false, R not strict max
$\neg P \vee S^{*}$	$\emptyset$	$\emptyset$	true
$R \vee S^{*}$	$\emptyset$	$\{S\}$	false

$N_I = \{S\}$

$N_I \not\models Q \vee R \vee R$

+ is selected

$Q \vee R \vee R \Rightarrow_{Sup} Q \vee R$

	N_D	S_D	
$\neg P^+ \vee Q^*$	$\emptyset$	$\emptyset$	true
$Q \vee R^*$	$\emptyset$	$\{R\}$	false R strictmax
$Q \vee R^* \vee R^*$	$\{R\}$	$\emptyset$	true
$\neg P \vee S^*$	$\{R\}$	$\emptyset$	true
$R \vee S^*$	$\{R\}$	$\emptyset$	true

saturated

$\neg P^* \vee \neg Q$ $\qquad$ $P > Q > R$

$P^* \vee R$

$\Longrightarrow$ Sup $\neg Q \vee R$

$\neg P^* \vee \neg Q^+$ $\qquad$ select $\neg Q$

$P^* \vee R$

no Sup inf possible, saturated

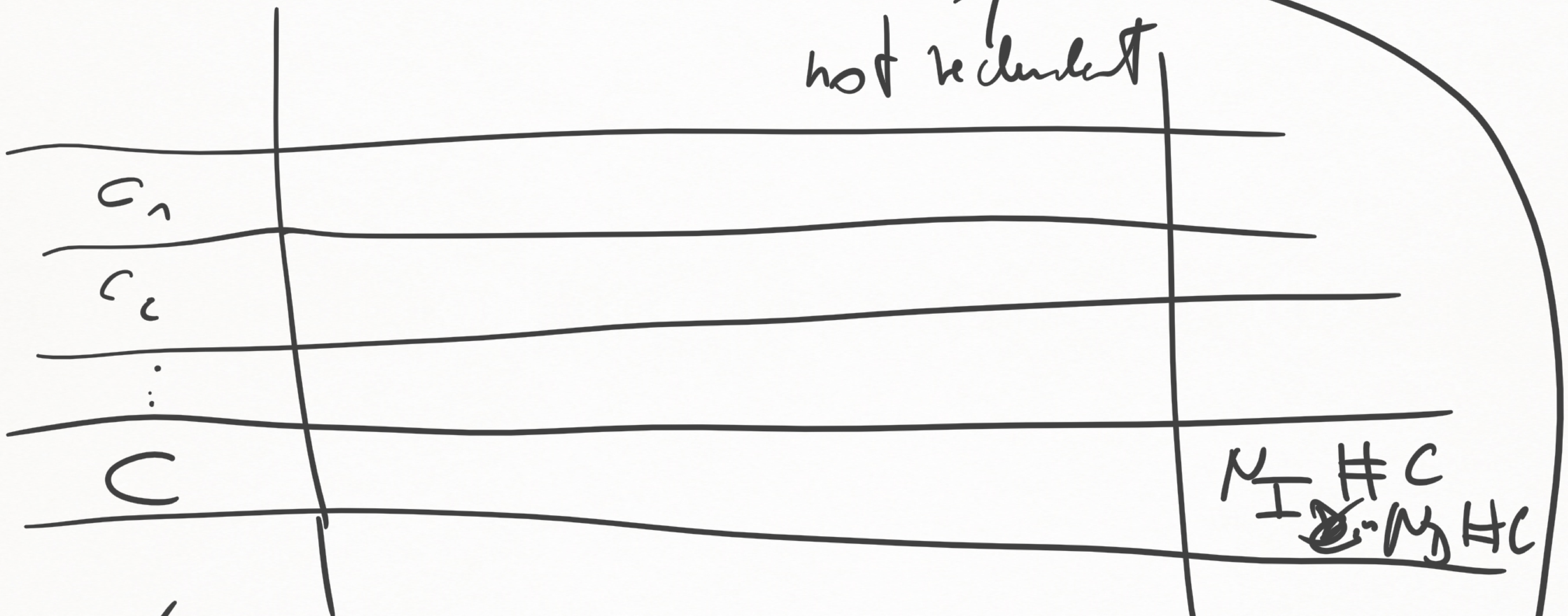
$M_I = \{P\}$

Completeness Proof:

assume N saturated $\bot \notin N$ such that $N_I \not\models N$

if $N_I \not\models N$ then there is a minimal $C \in N$, $N_I \not\models C$

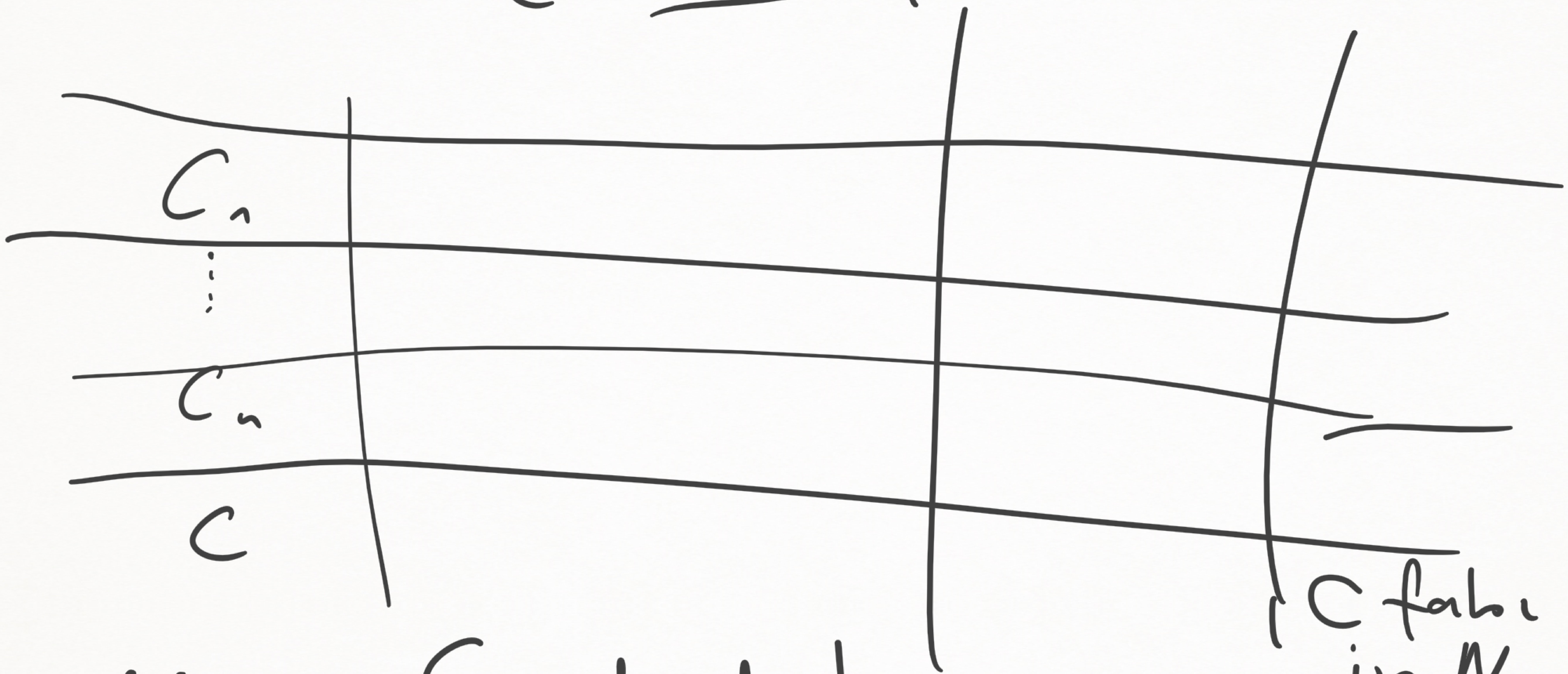
not redundant



$\cancel{S_C = \emptyset}$ $\quad C = C' \vee P^* \vee P^* \Rightarrow_{FAC} \underline{C' \vee P^*} \leftarrow C' \vee P^* \vee P^*$

not a max pos lit $C = C' \vee \neg P^* \quad P \in N_C$

C min false clause



assume C redundant

$$C_1,\dots,C_n \models C$$

because $N_C \models C_1,\dots,C_n$ — C min false clause!

$$\leadsto N_C \models C \ \lightning$$

Redundancy $N^{<C} \models C$

sat for SAT is NP-complete

non-redundancy is NP-complete

any clause set $N = \{C_1, \dots, C_n\}$

$N' = \{C_1 \lor P, C_2 \lor Q, C_3, \dots, C_n, \neg P, \neg Q\}$

P, Q fresh $\quad$ N is sat iff N' is sat

N' is sat $\quad$ $N' \setminus \{\neg P, \neg Q\} \not\models P \lor Q$

$N \not\models P \lor Q$ iff $N \cup \{\neg P, \neg Q\}$ is unsat

P, Q max all literals

Clauses generated/learned by CDCL are
<u>not</u> redundant

Effort learning a clause in CDCL is linear
in $|N|$

<u>Ordering</u> : fail!

$([K_1, \ldots, K_n], --)$ $K_1 < K_2 < \ldots$

learned not redundant, same proof

$(\underset{\text{regular}}{\underline{\qquad\qquad}}, \underset{\text{run}}{\underline{D \vee L}}) \Longrightarrow_{\text{CDCL}}^{\text{Backtrack}}$ ✓