One Sort

N monadic predicate

$$\underline{N(0)}$$

$$\underline{\wedge \forall x\, (N(x) \to N(s(x)))}$$

$$\mathcal{A}(N) \supseteq \{0, s(0), s(s(0)), \ldots\}$$

$$\text{plus}(0, b) \approx 0$$

$$\forall x, y\, [\text{plus}(s(x), y) \approx s(\text{plus}(x, y))]$$

$$\forall x\, (\text{plus}(0, y) \approx y)$$

one sort

$$\left[\exists x \; \forall x \, (P(x) \to P(f(x))) \wedge P(a)\right] \to P(f(a))$$

Valid

if premise of $\to$ is false then ✓

lets assume it holds

$\mathcal{A} \models P(a)$ $\quad$ $\mathcal{A} \models \forall x \, (P(x) \to P(f(x)))$

$a^{\mathcal{A}}$

$\mathcal{A} \models P(f(a))$ $\quad$ satisfiable

$\mathcal{A}(\beta[x \to a^{\mathcal{A}}]) \models (P(x) \to P(f(x)) \wedge P(a)) \to P(f(a))$

$\mathcal{A}(\beta[x \to b^{\mathcal{A}}])$ s.t. $\mathcal{A}(f(b)) \notin P^{\mathcal{A}}$

$$[\exists x (P(x) \to P(f(\cancel{x}))) \wedge P(a)] \to P(f(a))$$

satisfiable

assign for x different $a^{\mathcal{A}}$

$$\exists x [[P(x) \to P(f(x))] \wedge P(a)] \to P(f(a))$$

valid

$$\neg(\exists x (P(x) \to \cancel{P(f(x)))} \wedge P(a)) \vee P(f(a))$$

$$\exists x (\neg(P(x) \to P(f(x)) \wedge P(a)) \vee P(f(a))$$

$\forall x \, [P(x) \land \neg P(x)]$ unsat

$\forall x \, [P(x) \lor \neg P(x)]$ valid

$I3[x \mapsto u]$ total, no empty sets

$u \in U_A$ $u \in P^A$ or $u \notin P^A$

$\exists x \, [P(x) \land \neg P(x)]$ unsat

$\exists x \, [P(x) \lor \neg P(x)]$ vallid

$$\underline{(\forall x\, P(x))} \to (\exists y\, P(y))$$

false ✓ valid

$$\mathcal{A} \models \forall x\, P(x)$$

$$(\exists x\, P(x)) \to (\forall y\, P(y))$$

$\to P^{\mathcal{A}} = \emptyset \leadsto \mathcal{A} \not\models \exists x (P(x))$

$\to P^{\mathcal{A}} = U_{\mathcal{A}}$ $\mathcal{A}(\exists x\, P(x))$ iff $\mathcal{A}(\forall x\, P_x)$

$$\exists x \left[P(x) \to \forall y\, P(y) \right]$$

$$\mathcal{A} \nearrow P^{\mathcal{A}} \neq U_{\mathcal{A}}, \text{ i.e. } b \in U_{\mathcal{A}},\ b \notin P^{\mathcal{A}}$$

$$x \rightsquigarrow b$$

$$\mathcal{A} \searrow P^{\mathcal{A}} = U_{\mathcal{A}} \rightsquigarrow \mathcal{A} \models \forall y (P(y))$$

$\rightsquigarrow$ CNF : $\forall x \left[P(x) \wedge \exists y \neg P(y) \right]$

$\rightsquigarrow_{x \to a} P(x) \wedge \neg P(a)$ "a" fresh

$\Longrightarrow_{RES} \bot$

$$\exists x \left[P(x) \to \forall y\, P(y) \right]$$

neg, CNF $\rightsquigarrow$ $P(x) \wedge \neg P(a)$

<u>Herbrand</u> : $U_A = \{a\}$

FOL Form Φ $\rightsquigarrow$ negation, CNF Φ'

Φ' only constants

$$\exists^* \forall^* \left[\ldots P(x), R(x, y) \right]$$

$$\forall x \left(N(x) \to N(s(x)) \right) \wedge N(0)$$

<u>Herbrand</u> : $U_A = \underline{\{0, s(0), s(s(0)), \ldots\}}$